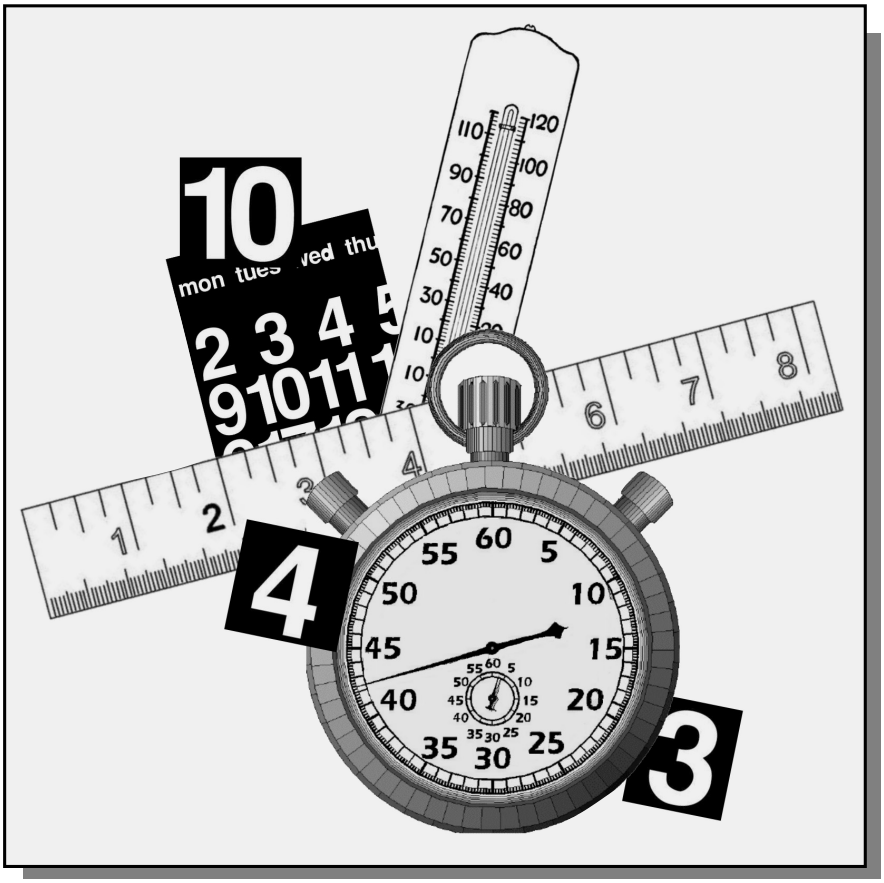
Basic Math Skills



Chapter 4

BMS: 76026 Revision: 030208

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Basic Math Skills

Chapter 4

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"The study of Mathematics cannot be replaced by any other activity that will train and develop man's purely logical faculties to the same level of rationality."

—C. O. Oakley



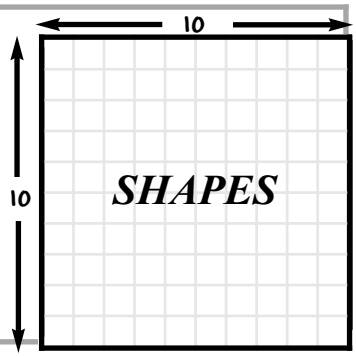
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Basic Math Skills

Lesson 1

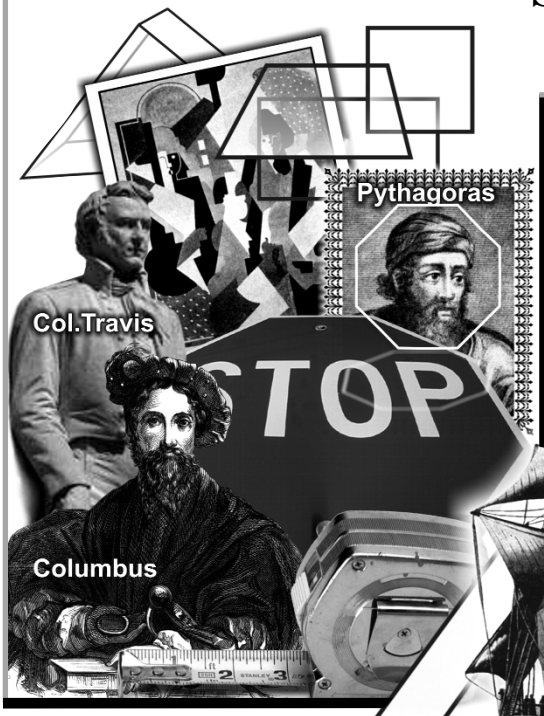
Geometry

Chapter 4



Section 1

Lessons 1-5



Men have used shapes throughout history for various reasons. From the beginning of time, families and countries have used shapes for crests and flags. Certain shapes on flags identify countries and families. Sailors have used various shapes and colors on flags to send messages to other ships. Men, such as Colonel Travis at the Alamo, have drawn ***lines*** in the sand to issue challenges of bravery and courage. Columbus understood that the world was round by reading Hebrew writings that spoke of the earth being in the shape of a circle.

You come in contact with shapes everyday. While driving you look at road signs in the shapes of squares, rectangles, triangles, and octagons.





VOCABULARY

Line: drawn between any two points and extended through those points indefinitely
Point: a location; the place where two lines intersect; has no dimensions; represented by a dot and labeled with a capital letter
Angle: two rays containing the same end point; or formed when two lines intersect
Line segment: the portion of a line that lies between two points
Endpoint: the point at the end of a line segment or ray

Look around at the natural things of this universe. The Sun and Moon are "circles" of light in the sky. Tree trunks are in the shape of round cylinders. Some spiders spin webs (lines of silk) in circular shapes. You can probably think of many more examples of various shapes in the world around you.

You may have heard that the shortest distance between two ***points*** is a straight line. Highway engineers know this fact and attempt to build roads and railroads accordingly. However, terrain in certain areas often requires a road to curve or even make a sharp turn (***angle***). Engineers also understand that when building a structure, the framework of the structure must be crisscrossed with triangles to add stability.

In ancient Egypt and Greece, shapes were of such importance that people believed everything could be explained

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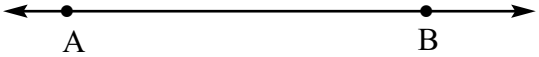
with lines, squares, and angles. These ancient cultures attempted to explain their gods by defining the universe in terms of *squares*. Priests in the temples of these cultures were required to know geometry so they could attempt to explain nature. Ancient cultures in Egypt built pyramids of squares and triangles for the tombs of Pharaohs. During the Renaissance (14th to 16th centuries), artists realized that certain shapes were pleasing to the eye, and were sketched and painted accordingly. Many paintings from that time period utilized squares, rectangles, and lines in ingenious ways to create a desired three-dimensional effect.

Historical emphasis on shapes demonstrates the importance of knowing about geometric shapes. Therefore, this chapter will introduce you to geometric shapes in their various forms and explain how to determine their perimeter, area, volume, and size of the angles (if any) they contain. As in construction, a building must have a strong foundation in order to stand. This lesson will consist primarily of definitions. This will be the foundation for future lessons on geometric shapes, so be sure to learn these basic definitions for geometry. Most geometric shapes, except those made from circles, consists of lines or ***line segments***. Study the definitions to know about lines and line segments.

Definition: Line

A line is drawn between any two points and extends indefinitely through the two points. A line consists of an infinite number of points.





- State boundaries along longitude and latitudes are lines.
- The equator is a line that crosses many national borders.

Vocabulary

Intersect: to cross or meet
Parallel: never intersecting
Point of intersection: the point at which lines intersect or cross
Transversal: a line that intersects two or more parallel lines
Ray: a part of a line; has an end point and extends in a straight line indefinitely
Right angle: an angle with a measurement of exactly 90°
Acute angle: an angle with a measurement of less than 90°
Obtuse angle: an angle with a measurement of more than 90°
Straight angle: an angle that measures 180°

Definition: Line Segment

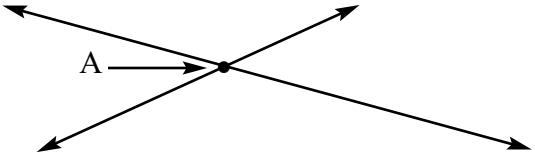
A line segment is the portion of a line between two points. Points A and B are called ***endpoints***.



- A dining room wall is a segment of the side of a house.
- A fence can be a segment of a property boundary.

Definition: Point

A point is the place where two lines ***intersect***. "A" is a point.

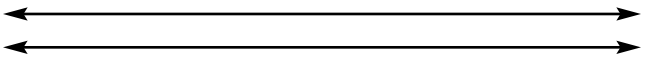


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- Two state highways intersect at a common point.
- A river and road may intersect in a valley.

Definition: Parallel Lines

Two or more lines are ***parallel*** if the lines have no ***point of intersection*** (the point at which lines intersect or cross). To run parallel to something means to run along with or beside, never crossing paths.

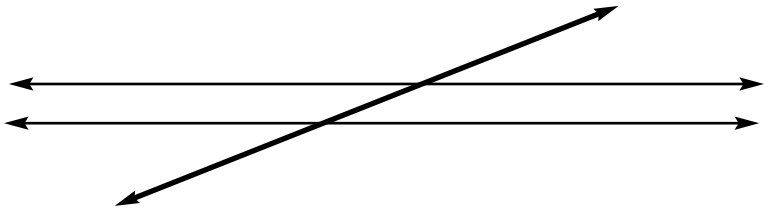


- Each side of a highway or railroad is parallel to the other side.
- The opposite edges of a rectangular table are parallel.

Definition: Transversal

A line that crosses or lies transverse across two or more parallel lines is called a ***transversal***.

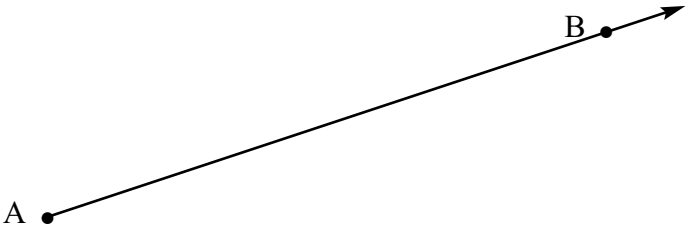




- To be *transverse* means to lie across, as a road could lie transverse to a highway.

Definition: Ray

A ***ray*** is a part of a line. A ray consists of an endpoint, A, and extends indefinitely in only one direction through another point, B.



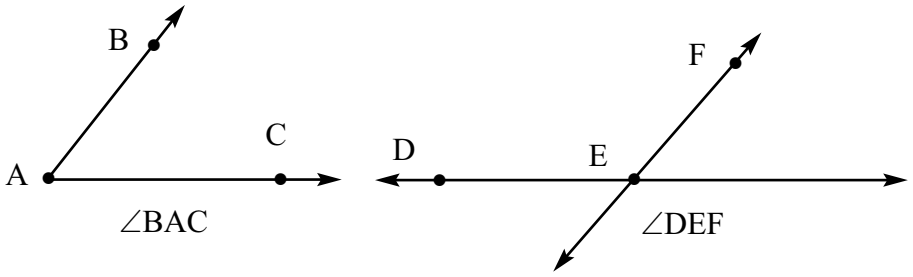
- You have probably heard of a ray of light from the sun. This ray originates at one point and continues in one direction indefinitely.
- Some people talk of having a ray of hope. Their hope lies in a "line" from one source or point.

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As stated earlier, most geometric shapes are formed by lines or line segments. When lines or line segments intersect, they form angles. The following definitions will familiarize you with the names of the various types of angles.

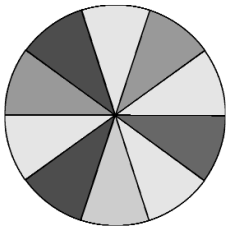
DEFINITION: ANGLE

An angle is formed by two rays containing the same endpoint. An angle is also formed by two intersecting lines. An angle is named using three points.

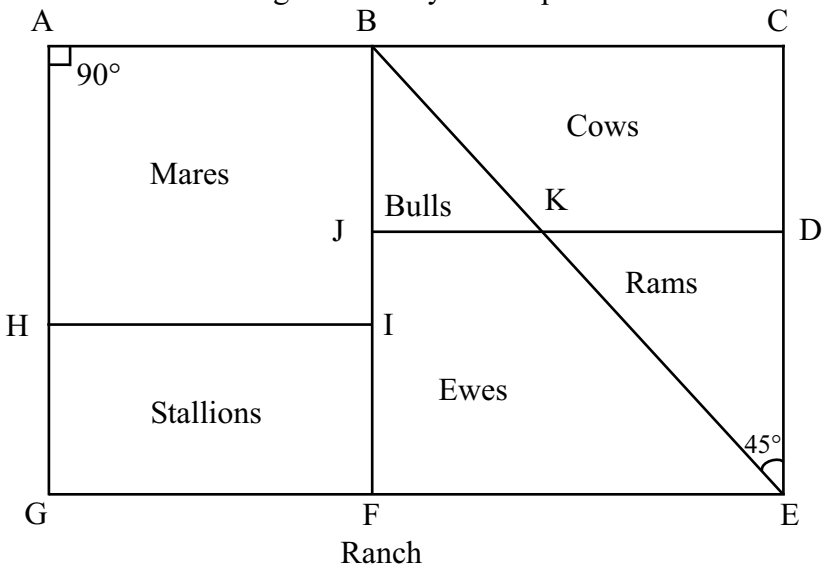


The symbol ∠ is used to indicate an angle. Three points name the angle. The three points are listed with one point on each side of the angle and the middle point is the point that is common to both lines.

- A pie cut into wedge-shaped pieces consists of angles formed by numerous intersecting lines (cuts of the pie).



- A ranch with intersecting fences may define pastures with various angles.

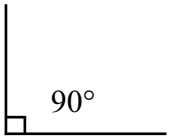


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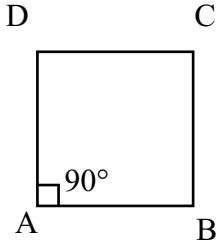
DEFINITION: RIGHT ANGLE

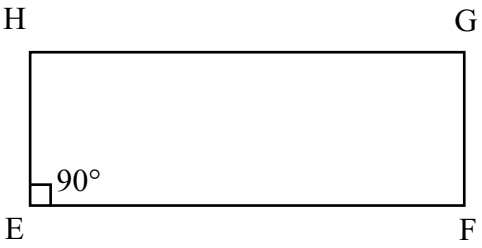
A ***right angle*** is an angle that has a measurement of 90°.





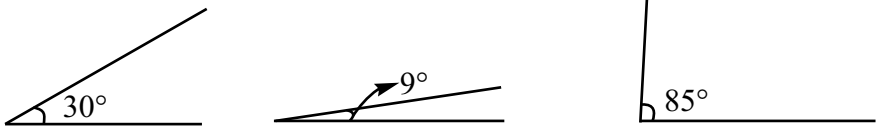
- Squares and rectangles are formed by four 90° angles or *right* angles.





DEFINITION: ACUTE ANGLE

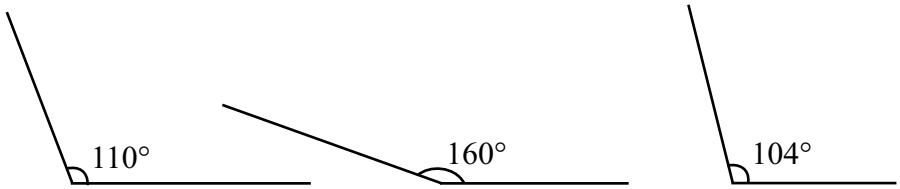
An ***acute angle*** is an angle that has a measurement of less than 90°.



- Pyramids contain acute angles over a square.

DEFINITION: OBTUSE ANGLE

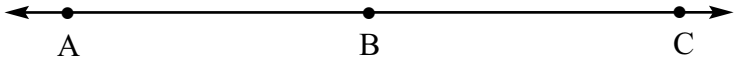
An ***obtuse angle*** is an angle that has a measurement of more than 90°.



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DEFINITION: STRAIGHT ANGLE

A ***straight angle*** is an angle that measurements 180°. A straight angle appears to be a straight line. ∠ABC is a straight angle.



Apply what you have learned about the definitions of angles by working the activities for this lesson.

LIFE PRINCIPLE



"The best formula for success is the 'try'-angle."

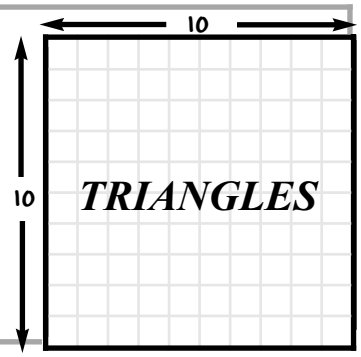
—Anonymous

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Lesson 2

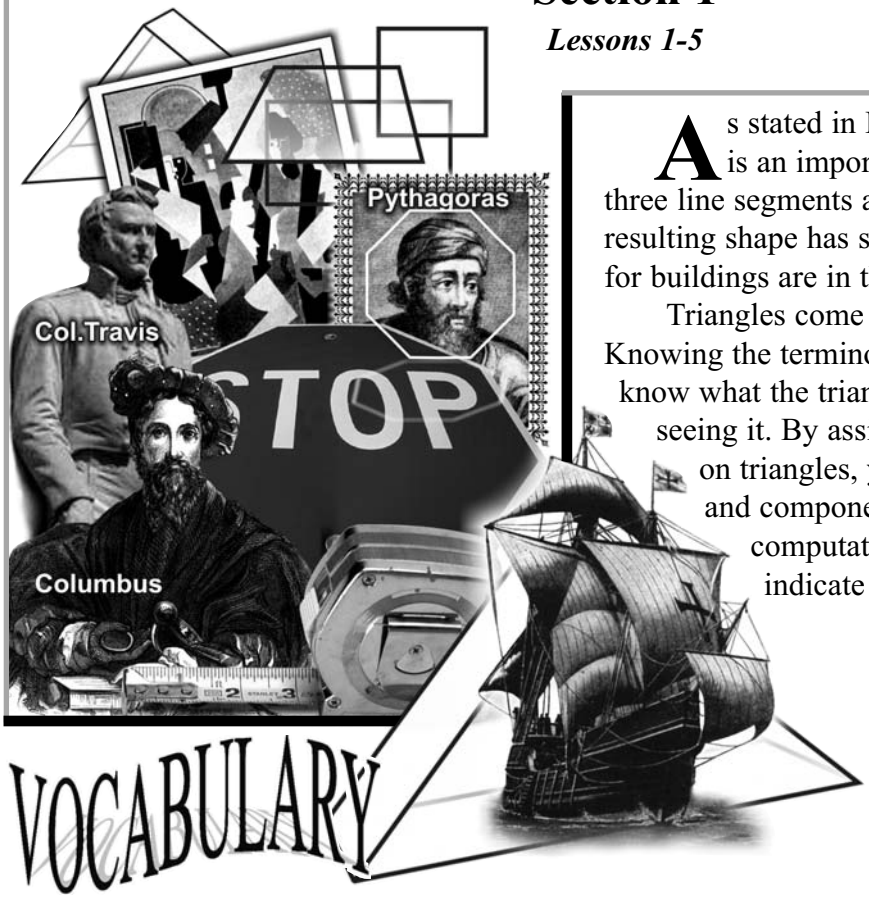
Geometry

Chapter 4



Section 1

Lessons 1-5



As stated in Lesson 1 of this chapter, a ***triangle*** is an important shape in mathematics. When three line segments are joined to form a triangle, the resulting shape has stability. That is the reason braces for buildings are in the shape of triangles.

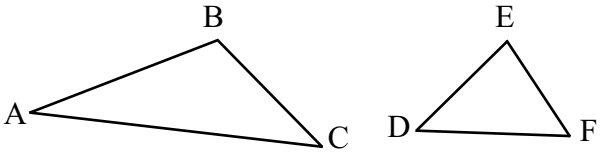
Triangles come in various shapes and sizes. Knowing the terminology of triangles will help you know what the triangle might look like without actually seeing it. By assigning alphabetical letters to points on triangles, you can easily visualize the shape and components with which to make computations. The symbol, $\triangle$, is used to indicate a triangle.

VOCABULARY

Triangle: a polygon with three sides
Right triangle: a triangle containing a right angle
Obtuse triangle: a triangle containing an obtuse angle
Acute triangle: a triangle containing three acute angles
Isosceles triangle: a triangle with two sides and two angles equal
Equilateral triangle: a triangle with all three sides and all three angles equal

DEFINITION: TRIANGLE

A triangle is a geometric figure with three sides and three angles. $\triangle ABC$ and $\triangle DEF$ below are examples of triangles.



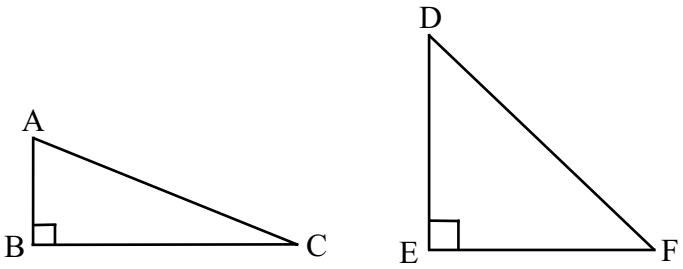
Study the following definitions to become familiar with the names and characteristics of special case triangles.

DEFINITION: RIGHT TRIANGLE

A ***right triangle*** is a triangle containing a right angle. Following are examples of right triangles.

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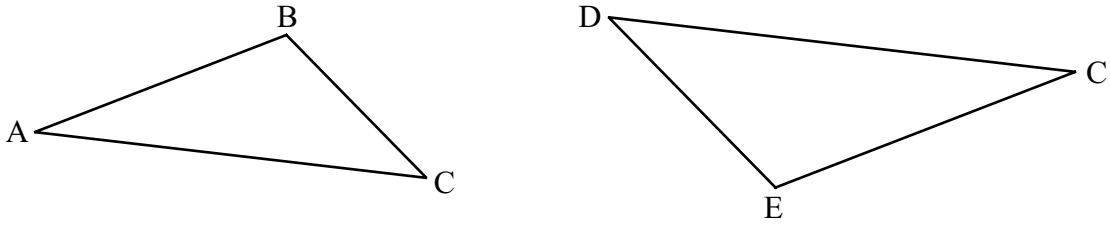
The measures of ∠B and ∠E equal 90 degrees.



DEFINITION: OBTUSE TRIANGLE

An ***obtuse triangle*** is a triangle that contains an obtuse angle. △ABC and △DEF below are obtuse triangles. ∠B and ∠E are obtuse angles (greater than 90°).





DEFINITION: ACUTE TRIANGLE

An ***acute triangle*** is a triangle containing three acute angles. △ABC and △DEF below are examples of acute triangles.



VOCABULARY

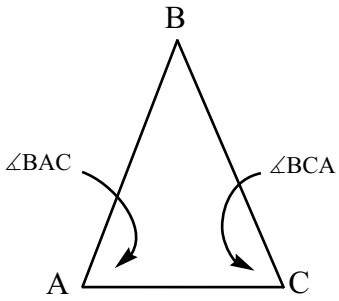
Vertex: the point of intersection of two sides of an angle
vertices: plural of vertex

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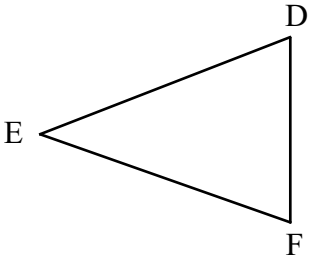
Definition: Isosceles Triangle



An ***isosceles triangle*** is a triangle with two sides of equal length and two angles of equal measure. $\triangle ABC$ below is an isosceles triangle. The sides of $\triangle ABC$, AB and BC are of equal length, and $\angle BAC$ and $\angle BCA$ are of equal measure.

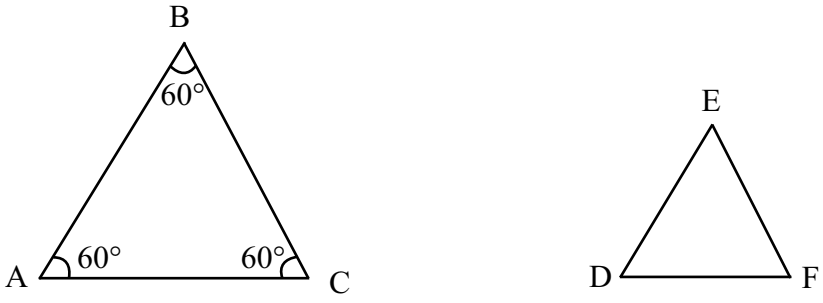


$\triangle DEF$ is another example of an isosceles triangle. The sides of $\triangle DEF$, DE and EF are of equal length, and $\angle EDF$ and $\angle EFD$ are of equal measure.



Definition: Equilateral Triangle

An ***equilateral triangle*** is a triangle in which all three sides are of equal length and all three angles are of equal measure. $\triangle ABC$ below is an equilateral triangle. The length of the line segments AB, BC, and CA are equal, and $m\angle ABC = m\angle BCA = m\angle CAB$.



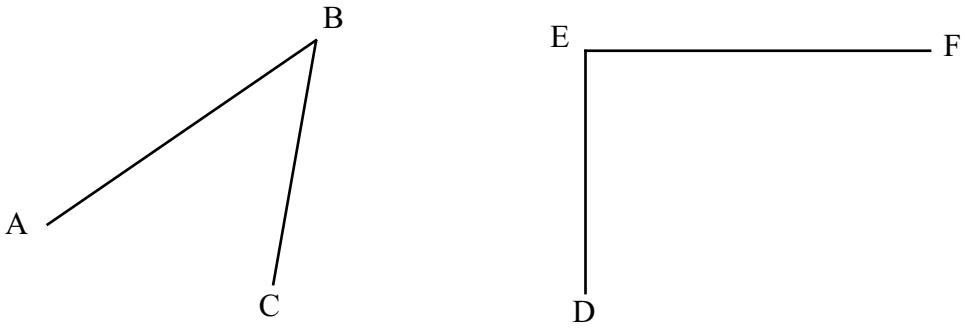
$\triangle DEF$ above is also an equilateral triangle. The length of the line segments DE, EF, and FD are equal, and $m\angle DEF = m\angle EFD = m\angle FDE$.

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DEFINITION: VERTEX

The ***vertex*** is the point of intersection of the sides of an angle.



The points B and E in the angles above are the ***vertices*** of ∠ABC and ∠DEF, respectively.

Apply what you have learned about the definitions of triangles by working the activities for this lesson.

LIFE PRINCIPLE



"A threefold cord is not easily broken."

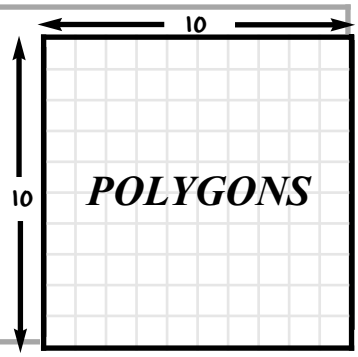
—Middle Eastern proverb

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Lesson 3

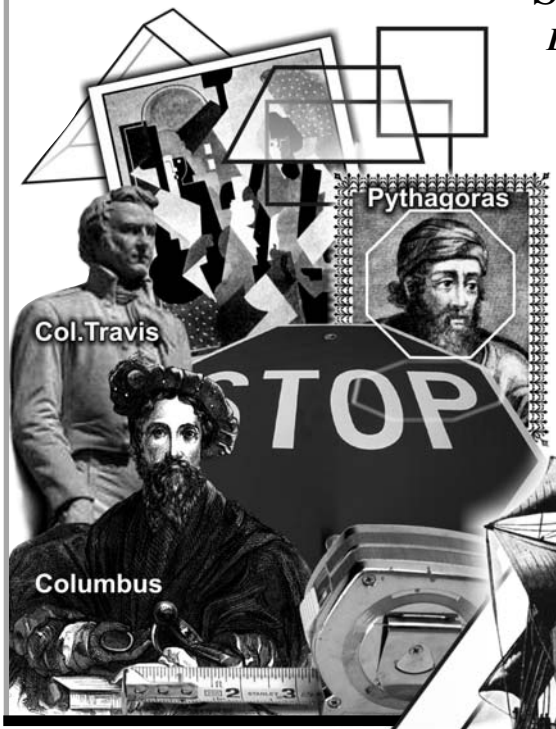
Geometry

Chapter 4



Section 1

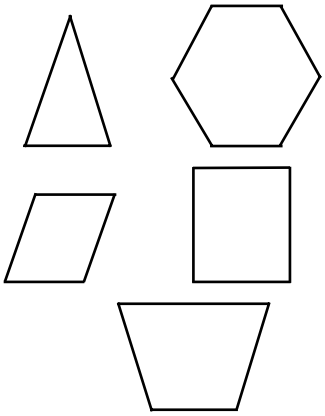
Lessons 1-5



A triangle is just one of many shapes in mathematics. A two dimensional shape with more than two sides is called a ***polygon***. Below is the mathematical definition of a polygon along with some examples.

DEFINITION: POLYGON

A polygon is a geometric figure with three or more sides. Each figure below is an example of a polygon.



VOCABULARY

Polygon: a two dimensional geometric figure with three or more sides; named according to the number of sides
Parallelogram: a polygon with four sides of which opposite sides are parallel
Rectangle: a parallelogram whose angles are all 90°
Square: a rectangle with all four sides equal

Polygons are named according to the number of sides of the polygon. **Table 3.1** shows the prefixes, the meaning of the prefix, the name of the polygon, and the number of sides of the polygon. The prefixes are derived from Latin roots for the numbers represented. By memorizing the meaning of the prefixes you can immediately visualize how many sides the polygon contains.

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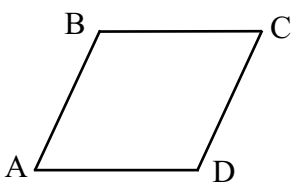
Table 3.1

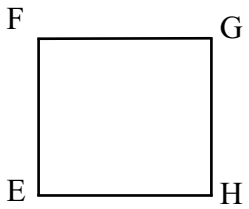
Prefix	Meaning	Name	Sides/Angles
tri	3	Triangle	3
quadri	4	Quadrilateral	4
penta	5	Pentagon	5
hexa	6	Hexagon	6
hepta	7	Heptagon	7
octa	8	Octagon	8
nona	9	Nonagon	9
deca	10	Decagon	10

DEFINITION: PARALLELOGRAM



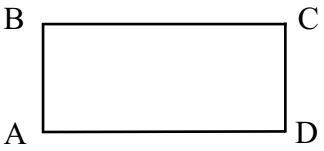
A ***parallelogram*** is a quadrilateral with opposite sides parallel. The wording, *is parallel to*, is symbolized as two short parallel lines, ||. The figure ABCD below is an example of a parallelogram. AB || CD and BC || AD. EFGH is also an example of a parallelogram. EF || GH and EH || FG.





DEFINITION: RECTANGLE

A ***rectangle*** is a parallelogram that contains a right angle. (All four angles of a rectangle are 90 degrees.)



- A sheet of copy paper is in the shape of a rectangle.
- The tops of most office desks are in the shape of a rectangle.

VOCABULARY

Circle: a geometric figure composed of points equidistance from a fixed point, called the center of the circle

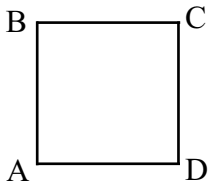
Trapezoid: a geometric figure with two parallel sides, called bases, and two nonparallel sides

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DEFINITION: SQUARE

A ***square*** is a rectangle with all sides of equal length.

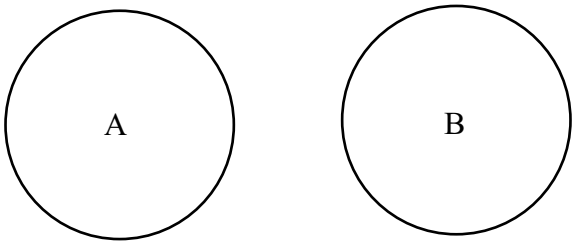




- The center area of most small towns is called a town square.
- Compact disc (CD) cases are in the shape of a square.

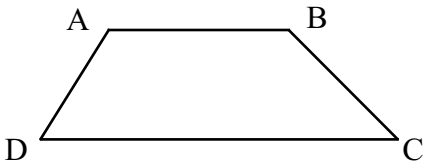
DEFINITION: CIRCLE

A ***circle*** is a set of points, equidistance from a fixed point. This fixed point is called the center of the circle. The name given the center of the circle, names the circle. Notice circles A and B below.



DEFINITION: TRAPEZOID

A ***trapezoid*** is a four-sided geometric shape containing two parallel sides and two non parallel sides. In the drawing of a trapezoid below, AB ∥ CD but AD is not parallel to BC.



- Some buildings are in the shape of a trapezoid.
- The sides of some waste baskets are in the shape of a trapezoid.

Apply what you have learned about the definitions of polygons and circles by working the activities for this lesson.

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Life Principle



"A chain is as strong as its weakest link."

—Anonymous

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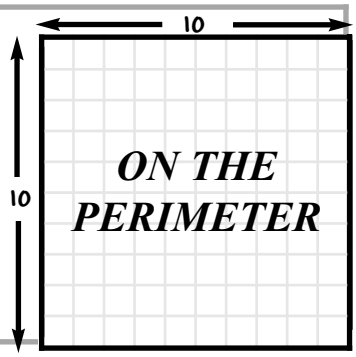
Lesson 4

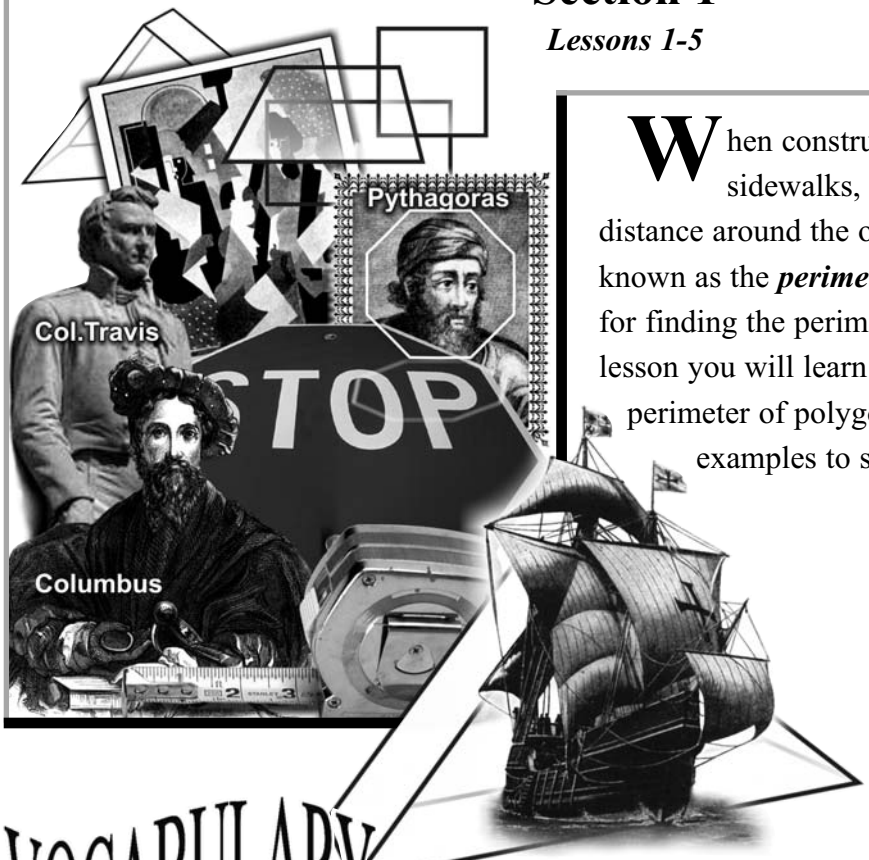
Geometry

Chapter 4

Section 1

Lessons 1-5





When constructing buildings, fences, and sidewalks, builders need to know the distance around the object. This distance is known as the ***perimeter*** and certain rules apply for finding the perimeter of various shapes. In this lesson you will learn formulas for finding the perimeter of polygons and circles. Study the examples to see how to use the formulas.

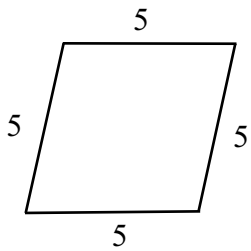
DEFINITION: PERIMETER

The perimeter of a polygon is determined by finding the sum of the length of all the sides.

VOCABULARY

Perimeter: distance around the outside of something
Radius: a line segment between the point that is the center of a circle and a point on the circle
Diameter: a line segment whose end points are on the circle, with the center of the circle being a point on the line; the distance across a circle
Circumference: the distance around, or perimeter of a circle

Example Set 1



In the parallelogram above, each side is 5 units. The perimeter is calculated by adding the length of each side together. Therefore, the result is the following equation.

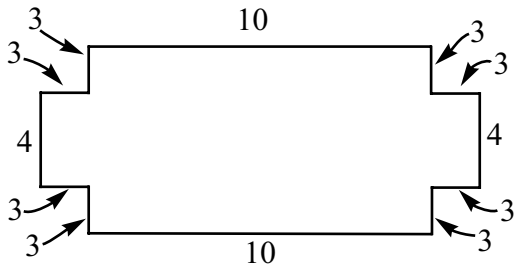
perimeter = 5 + 5 + 5 + 5 = 20 units

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Example Set 2

Look at the figure below and follow the steps for calculating the perimeter.





The figure above has twelve sides. The perimeter is calculated by adding all twelve sides together. The result is the following equation.

perimeter = 10 + 3 + 3 + 4 + 3 + 3 + 10 + 3 + 3 + 4 + 3 + 3 = 52 units

This figure is symmetrical which means that one half of the figure is the mirror image of the other half. Therefore, the measurements of opposite corresponding sides will be the same.

FORMULA FOR PERIMETER

The formula for calculating the perimeter, P, of a rectangle is two times the width, w, plus two times the length, l. The equation for the perimeter of a rectangle is shown below.

$$P = 2w + 2l$$

Example Set 3

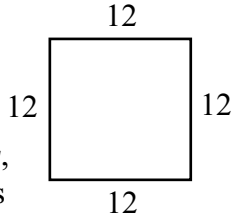
Find the perimeter of a rectangle with a width of 5 inches and a length of 7 inches. To obtain the answer to this problem you simply "plug in" or substitute the numeric value of each dimension into the formula and solve for the perimeter. The width, $w = 5$ and the length, $l = 7$. Therefore the formula becomes:

$$P = 2(5) + 2(7) = 10 + 14 = 24 \text{ inches}$$

The perimeter of this rectangle is 24 inches. Note: You may find it useful to draw a picture so that you can see what is happening while you solve the problem. Do not be afraid to draw a picture so the problem can be more easily solved.

Example Set 4

Aunt Polly wants to install a fence in a portion of her back yard. Aunt Polly wants the fenced area to be in the shape of a square, 12 feet on each side. Aunt Polly will have to purchase _____ yards of fence.



Draw a picture of the square area Aunt Polly wishes to fence and label each side as 12 feet. The formula for the perimeter of a square is $P = s + s + s + s$, because all the sides, (s) are equal. Notice that the units of the answer are in yards and the dimensions were given in feet. Therefore, since there are three feet in a yard, you must divide the dimensions or the answer by three. In this example we will divide the dimensions by three, first.

12 feet ÷ 3 feet/yard = 4 yards

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Therefore, each side of Aunt Polly's fence will be 4 yards long. The perimeter of the square would be $P = 4 + 4 + 4 + 4 = 16$ yards. Aunt Polly would have to purchase 16 yards of fence to enclose a square area 12 feet on each side.

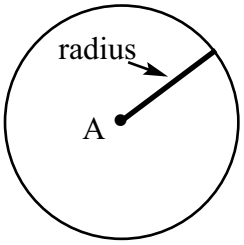
The next shape that we will learn how to find the distance around is a circle. You will need to know two definitions that are important in the formula for finding the circumference of a circle. These definitions are presented below.

DEFINITION: RADIUS

The ***radius***, *r*, of a circle is the distance from the center of the circle to a point on the circle.



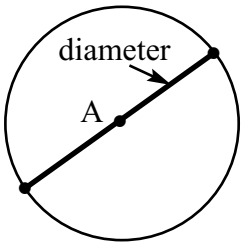
The line in the figure to the right is the radius of circle A.



DEFINITION: DIAMETER

The ***diameter***, *d*, of a circle is the distance across a circle. The diameter is the length of a line that passes through the center of the circle and whose endpoints lie on the circle.

The line in the figure to the right is the diameter.



FORMULA FOR CIRCUMFERENCE

The formulas for finding the ***circumference***, *C*, of a circle are: $C = 2\pi r$, where $\pi = 3.14$ (approximation) and *r* is the radius of the circle. $C = \pi d$, where *d* is the diameter of a circle. The symbol, π, is pronounced "pie."

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Example Set 5

Look at the following explanation to find the circumference of a circle with a radius of 1 inch and diameter of 2 inches.

First, using the formula for circumference that includes r for radius we have: $C = 2\pi r = 2(3.14)(1) = 6.28(1) = 6.28$ inches.

Next, using the formula for circumference of a circle with a diameter, 2 inches, we have: $C = \pi d = \pi(2) = 3.14(2) = 6.28$ inches.

Apply what you have learned about the perimeters of geometric shapes by working the activities for this lesson.

LIFE PRINCIPLE



"Truth is straight;
lies go in circles."

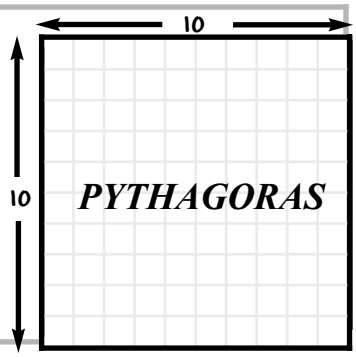
—Anonymous

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Lesson 5

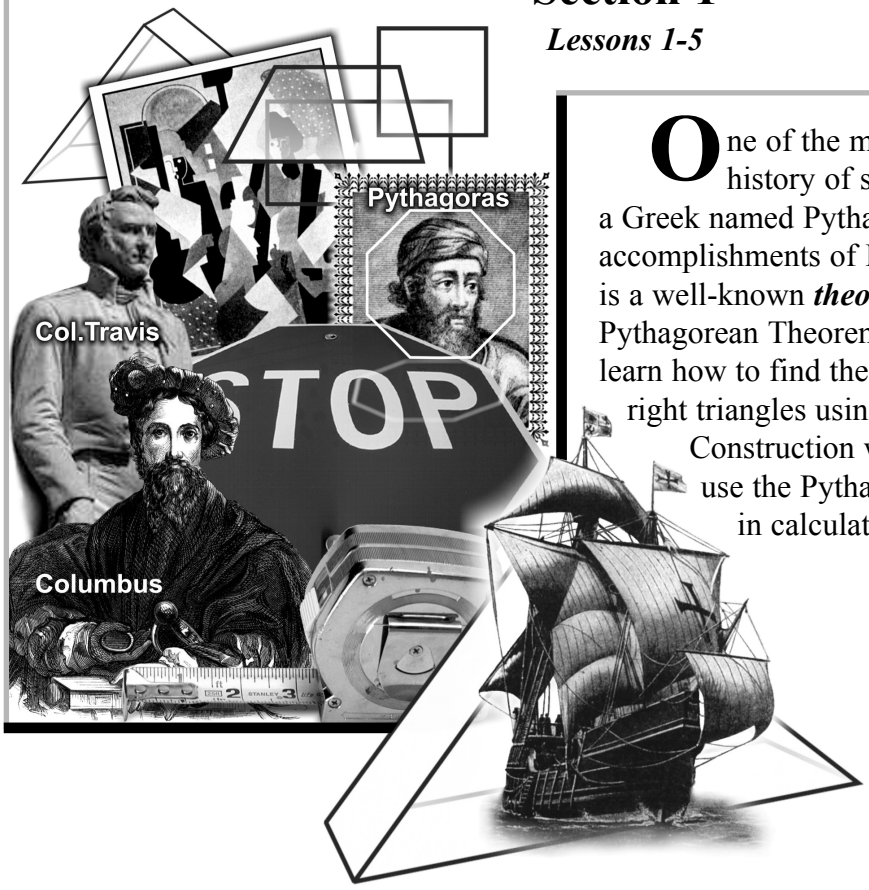
Geometry

Chapter 4



Section 1

Lessons 1-5



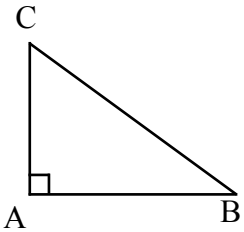
One of the most important men in the history of science and mathematics was a Greek named Pythagoras. Among the accomplishments of Pythagoras and his followers is a well-known ***theorem*** in geometry, the Pythagorean Theorem. In this lesson, you will learn how to find the measurements of the sides of right triangles using the Pythagorean Theorem. Construction workers and builders often use the Pythagorean Theorem, especially in calculating measurements of roofs.

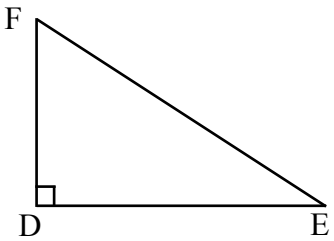
DEFINITION: HYPOTENUSE

The ***hypotenuse*** of a right triangle is the side opposite or across from the right angle (90°).

VOCABULARY

Theorem: a rule in math expressed by a formula or equation
Hypotenuse: the side opposite the 90° angle in a right triangle; the longest side of a right triangle
Radical form: form of a mathematical expression containing a radical sign



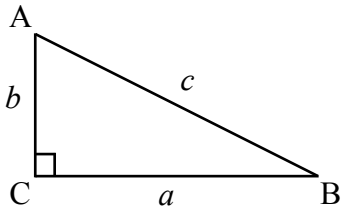


The line segment, BC, is the side of △ABC opposite the right angle. Therefore, BC is the hypotenuse of △ABC. The line segment, EF, is the side of △DEF opposite the right angle. Therefore, EF is the hypotenuse of △DEF.

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PYTHAGOREAN THEOREM

The Pythagorean Theorem states that in a right triangle the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides.



Let a, b, and c represent the lengths of the sides of a right triangle opposite the correspondingly labeled angles ∠A, ∠B, and ∠C. If ∠C is a right angle, then the following is true.

$$c^2 = a^2 + b^2$$

DEFINITION: SQUARE ROOT





The square root of a number is one of two equal factors of the number. To take the square root means to find those two equal factors. The radical sign, $\surd$, indicates the square root of the number under the radical sign, or the two equal factors of the number under the radical sign.

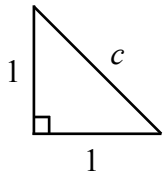
$\sqrt{4} = \sqrt{2 \bullet 2} = 2$; The square root of 4 is 2.

$\sqrt{9} = \sqrt{3 \bullet 3} = 3$; The square root of 9 is 3.

$\sqrt{25} = \sqrt{5 \bullet 5} = 5$; The square root of 25 is 5.

Example Set 1

Follow the steps below to learn how to use the Pythagorean Theorem to find the value of c in the right triangle shown in the figure.



In the right triangle above, a = 1 unit and b = 1 unit and c is the hypotenuse. Place the values of a and b in the formula for the Pythagorean Theorem as shown below.

Step 1 $c^2 = a^2 + b^2$ Step 1 states the formula for the Pythagorean Theorem.

Step 2 $c^2 = 1^2 + 1^2$ In Step 2, the values for a and b are placed in the formula.

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Step 3	$c^2 = 1 + 1$	In Step 3, the calculations, $1^2 = 1$, are performed and the answer placed in the appropriate position.
Step 4	$c^2 = 2$	In Step 4, $1 + 1 = 2 \rightarrow c^2 = 2$.
Step 5	$\sqrt{c^2} = \sqrt{2}$	The operation of square root is performed on each side of the equation in Step 5. (Remember, what you do to one side of the equation, you must do to the other.)
Step 6	$\sqrt{c^2} = c$ $c = \sqrt{2}$	Step 6 shows that $\sqrt{c^2} = c$, because $c \bullet c = c^2$. Therefore, $c = \sqrt{2}$

In **Example Set 1**, Step 6, the length of the hypotenuse, *c*, is $\sqrt{2}$ units. For simplicity, at this time, the answer in ***radical form***, $\sqrt{2}$, is considered the proper form since no two equal whole numbers multiply to obtain 2. In later lessons you will be instructed in the use of a calculator for calculating the decimal equivalent (a number containing a decimal point equal to a fraction) of radical expressions (expressions containing a radical sign).



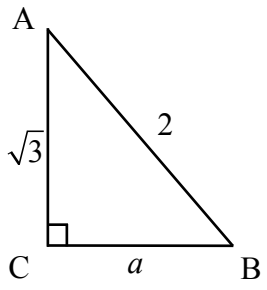
RULE

When a number in radical form is squared, the answer is the number under the radical sign.

$$(\sqrt{3})^2 = \sqrt{3} \bullet \sqrt{3} = 3$$
$$(\sqrt{2})^2 = \sqrt{2} \bullet \sqrt{2} = 2$$
$$(\sqrt{5})^2 = \sqrt{5} \bullet \sqrt{5} = 5$$

Example Set 2

Follow steps 1 - 6 to learn how to use the Pythagorean Theorem to find the value of *a* in the right triangle shown in the figure.



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In the right triangle of this Example Set, $c = 2$ units and $b = \sqrt{3}$ units and "a" is the unknown length of the shortest side. Place the values of c and b in the formula for the Pythagorean Theorem as shown below.

Step 1	$c^2 = a^2 + b^2$	Step 1 states the formula for the Pythagorean Theorem.
Step 2	$2^2 = a^2 + (\sqrt{3})^2$	In Step 2, the values for c and b are placed in the formula.
Step 3	$4 = a^2 + 3$	In Step 3, the calculations, $2^2 = 4$ and $(\sqrt{3})^2$, are performed and the answers placed in the appropriate position.
Step 4	$4 = 1 + 3$	In Step 4, $a^2 = 1$ since *1* is the only number that when added to *3* will equal *4*.
Step 5	$\sqrt{a^2} = \sqrt{1}$	From Step 4 we learned that $a^2 = 1$. Using this equation the operation of square root is performed on each side of the equation in Step 5. (Remember, what you do to one side of the equation, you must do to the other.)
Step 6	$\sqrt{a^2} = \sqrt{a}$ $a = \sqrt{1} = 1$	Step 6 shows that $\sqrt{a^2} = a$, because $a \bullet a = a^2$. Therefore, $a = 1$.

Apply what you have learned about the Pythagorean Theorem by working the activities for this lesson.

LIFE PRINCIPLE



"Any fact facing us is not as important as our attitude toward it, for that determines our success or failure."

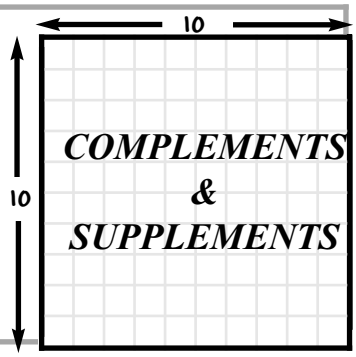
—Norman Vincent Peale (1898 - 1993)
Clergyman and writer

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Geometry

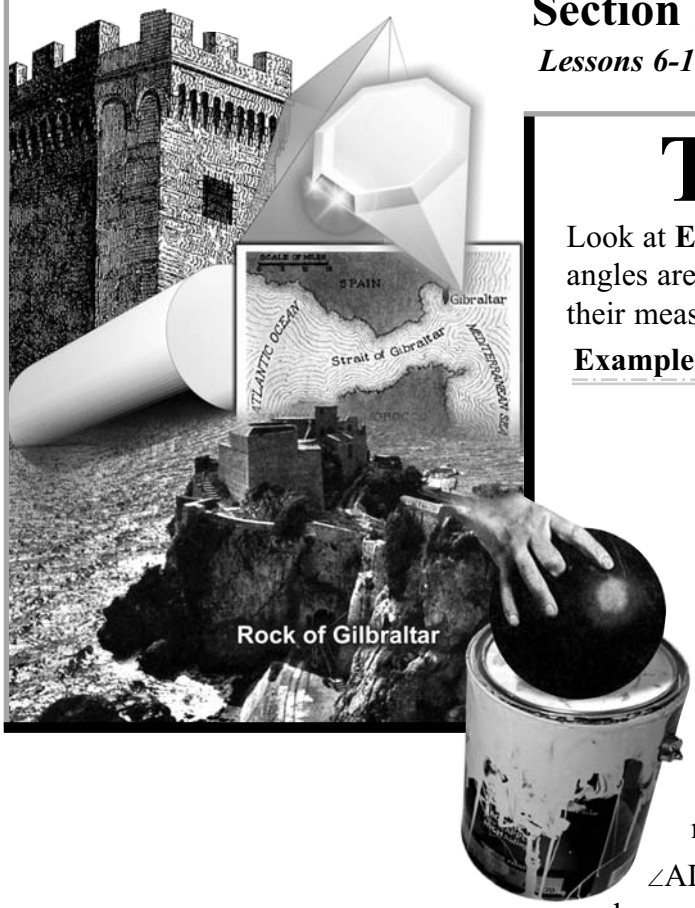
Lesson 6

Chapter 4



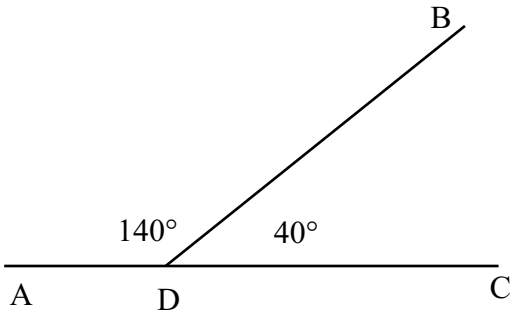
Section 2

Lessons 6-10



Two angles are ***supplementary angles*** if the sum of their measures equals 180°. Look at **Example Sets 1 and 2** to see how two angles are supplementary angles because the sum of their measures equal 180°.

Example Set 1



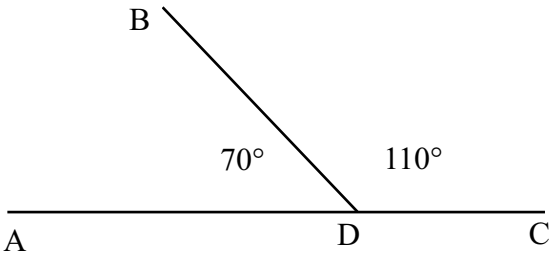
$m\angle ADB = 140°$

$m\angle BDC = 40°$

$m\angle ADB + m\angle BDC = 140° + 40° = 180°$

$\angle ADB$ and $\angle BDC$ are supplementary angles because the sum of their measures is 180°.

Example Set 2



$m\angle ADB = 70°$

$m\angle BDC = 110°$

$m\angle ADB + m\angle BDC = 70° + 110° = 180°$

$\angle ADB$ and $\angle BDC$ are supplementary angles because the sum of their measures is 180°.

VOCABULARY

Supplementary angles: angles whose sum of their measures equals 180°

m∠: the measure of an angle

Complementary angles: angles whose sum of their measures equals 90°

Vertical angles: angles on opposite sides of intersecting lines

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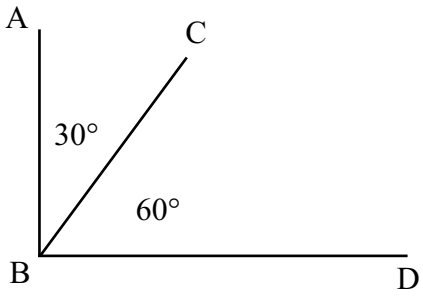


Definition: Complementary Angles

Two angles are ***complementary angles*** if the sum of their measures equals 90°.



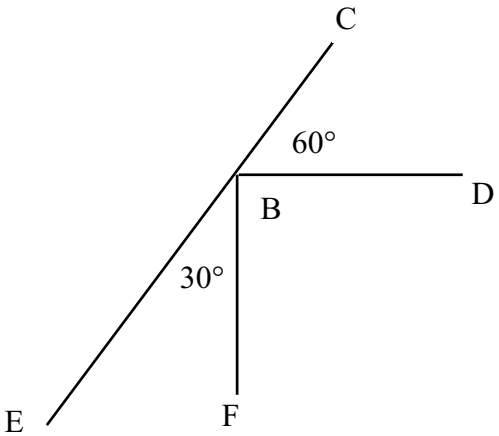
Example Set 3



$m\angle ABC = 30°$
$m\angle CBD = 60°$
$m\angle ABC + m\angle CBD = 90°$

$\angle ABC$ and $\angle CBD$ are complementary angles because the sum of their measures equals 90°.

Example Set 4



$m\angle EBF = 30°$
$m\angle CBD = 60°$
$m\angle EBF + m\angle CBD = 90°$

$\angle EBF$ and $\angle CBD$ are complementary angles because the sum of their measures equals 90°.

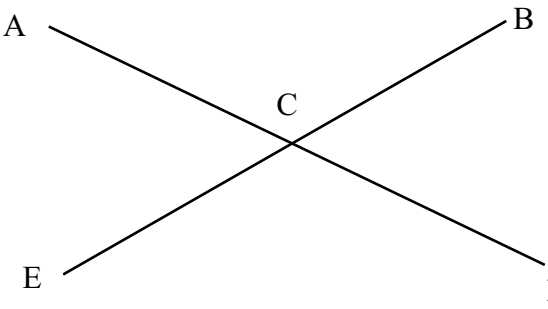


Definition: Vertical Angles

Angles formed opposite each other by two intersecting lines are ***vertical angles***. Vertical angles have equal measures.

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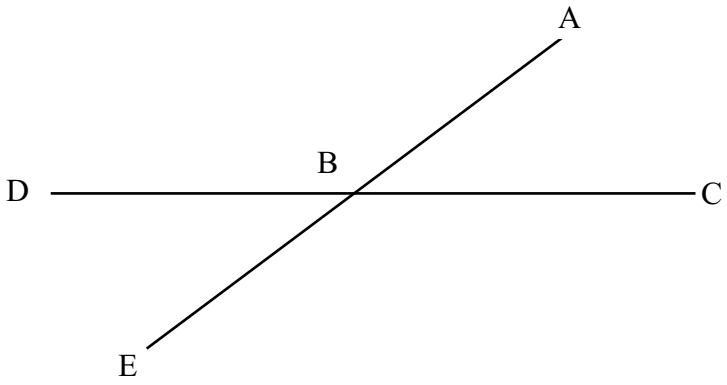
Example Set 5





∠ACE and ∠BCD are vertical angles, therefore, m∠ACE = m∠BCD.
∠ECD and ∠ACB are vertical angles, therefore, m∠ECD = m∠ACB.

Example Set 6



∠ABC and ∠DBE are vertical angles, therefore, m∠ABC = m∠DBE.
∠ABD and ∠EBC are vertical angles, therefore, m∠ABD = m∠EBC.

Demonstrate your understanding of angles by working the activities for this lesson.

LIFE PRINCIPLE



"People who work well together are complements to each other."

—Anonymous

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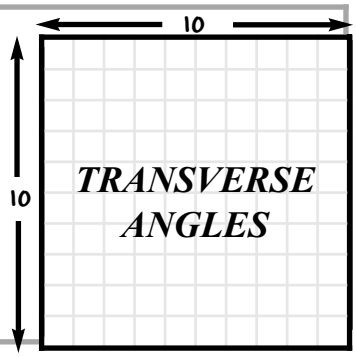
Lesson 7

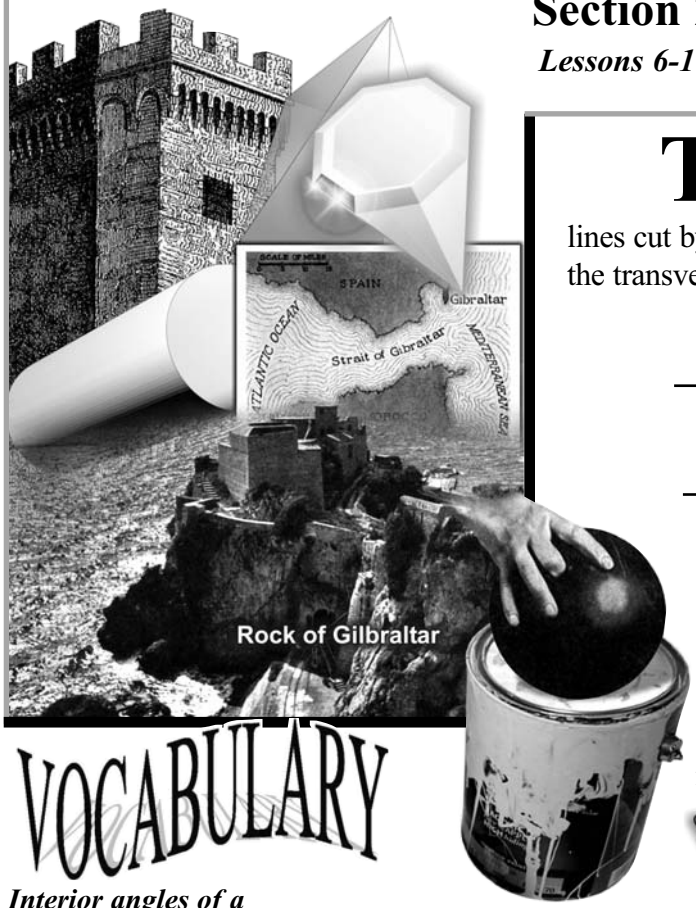
Geometry

Chapter 4

Section 2

Lessons 6-10





The ***interior angles of a transversal*** are the angles located between two parallel lines cut by the ***transversal*** and on the same side of the transversal; the sum of their measures equals 180°.

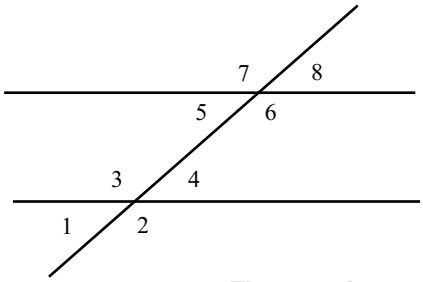


Figure 7.1

Look at **Figure 7.1**. ∠3 and ∠5; ∠4 and ∠6 are interior angles of a transversal. The sum of the measures of the interior angles is 180°; m∠3 + m∠5 = 180 and m∠4 + m∠6 = 180°

Interior angles of a transversal: angles between two parallel lines cut by a transversal and on the same side of the transversal; the sum of their measures equals 180°

Transversal: a line that intersects two or more parallel lines (Chapter 4, Section 1, Lesson 1)

Corresponding angles of a transversal: angles in a corresponding position in relationship to two parallel lines and a transversal of those two lines; corresponding angles are congruent



DEFINITION: CORRESPONDING ANGLES OF A TRANSVERSAL

The ***corresponding angles of a transversal*** are the angles located in the same positions in relationship to two parallel lines and a transversal of those two lines; corresponding angles are ***congruent***.

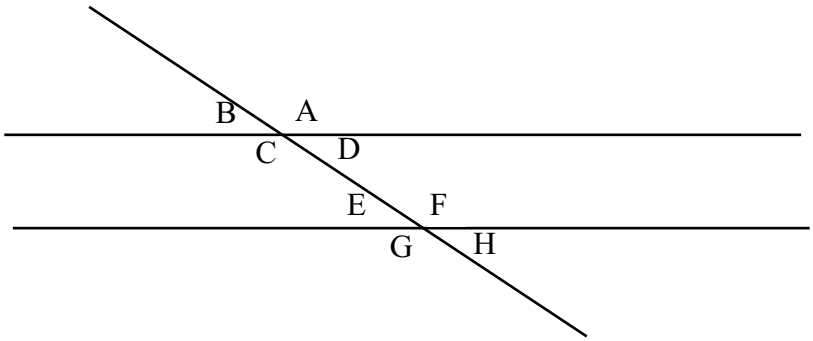
Look at **Figure 7.1** again. Corresponding angles in **Figure 7.1** are listed below and shown as congruent.

∠1 ≅ ∠5; ∠3 ≅ ∠7; ∠2 ≅ ∠6; ∠4 ≅ ∠8

Look at **Example Set 1** on the next page to see the relationship between the angles involved when a transversal intersects two parallel lines. You may need to refer to previous lessons to remember terms such as supplementary, complementary, and vertical angles.

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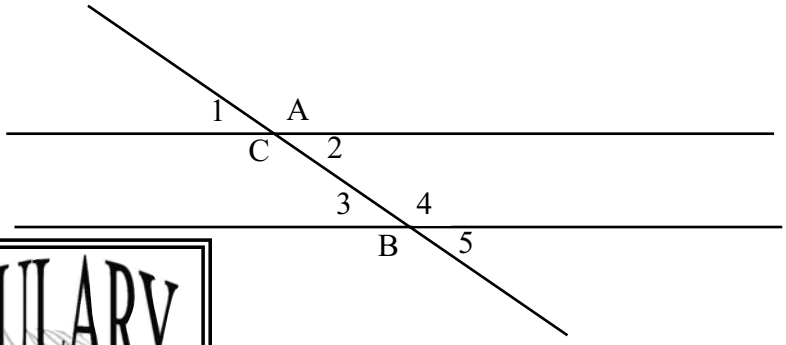
Example Set 1





Vertical Angles (congruent): $\angle B \cong \angle D$; $\angle A \cong \angle C$; $\angle E \cong \angle H$; $\angle F \cong \angle G$
Supplementary angles (sum of the measures equals 180°): $\angle A$ and $\angle D$; $\angle D$ and $\angle F$; $\angle F$ and $\angle H$; $\angle H$ and $\angle G$; $\angle G$ and $\angle E$; $\angle E$ and $\angle C$; $\angle C$ and $\angle B$; $\angle B$ and $\angle A$; $\angle C$ and $\angle D$; $\angle E$ and $\angle F$
Interior angles of a transversal (sum of the measures equals 180°): $\angle F$ and $\angle D$; $\angle E$ and $\angle C$
Corresponding angles of a transversal (congruent): $\angle D \cong \angle H$; $\angle A \cong \angle F$; $\angle B \cong \angle E$; $\angle C \cong \angle G$

Example Set 2



Notice the relationships between the angles shown.

Relationship of angles	Reason
$\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 5$	vertical angles
$\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 5$	corresponding angles
$\angle 4$ supplementary to $\angle 5$	By definition of supplementary angles, $\angle 4$ and $\angle 5$ form a straight angle.

VOCABULARY

Congruent, $\cong$: for angles, measure or size equal; for line segments, length equal
Alternate interior angles of a transversal: angles that are between two parallel lines cut by a transversal and on opposite sides of the transversal. Alternate interior angles are congruent.
Alternate exterior angles of a transversal: angles that are above and below two parallel lines cut by a transversal and on opposite sides of the transversal. Alternate exterior angles are congruent.

In **Example Set 2**, you can see how the relationship of alternate interior angles in the following definition can be stated.

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Definition: Alternate Interior Angles of a Transversal

Alternate interior angles of a transversal are angles that are between two parallel lines cut by a transversal and on opposite sides of the transversal. Alternate interior angles are congruent. $\angle 2 \cong \angle 3$, $\angle C \cong \angle 4$

Look again at **Example Set 2**. Notice how the relationship of alternate exterior angles in the following definition can be stated.

Definition: Alternate Exterior Angles of a Transversal

Alternate exterior angles of a transversal are angles that are above and below two parallel lines cut by a transversal and on opposite sides of the transversal. Alternate exterior angles are congruent. $\angle 1 \cong \angle 5$, $\angle A \cong \angle B$

Work the activities for this lesson to apply what you have learned about the definitions of angles formed by a transversal that cuts two parallel lines.

Life Principle



"Two wrongs do not make something right."

—Pioneer Axiom

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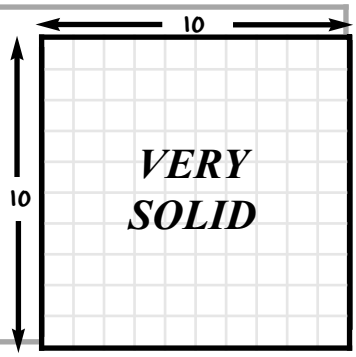
Lesson 8

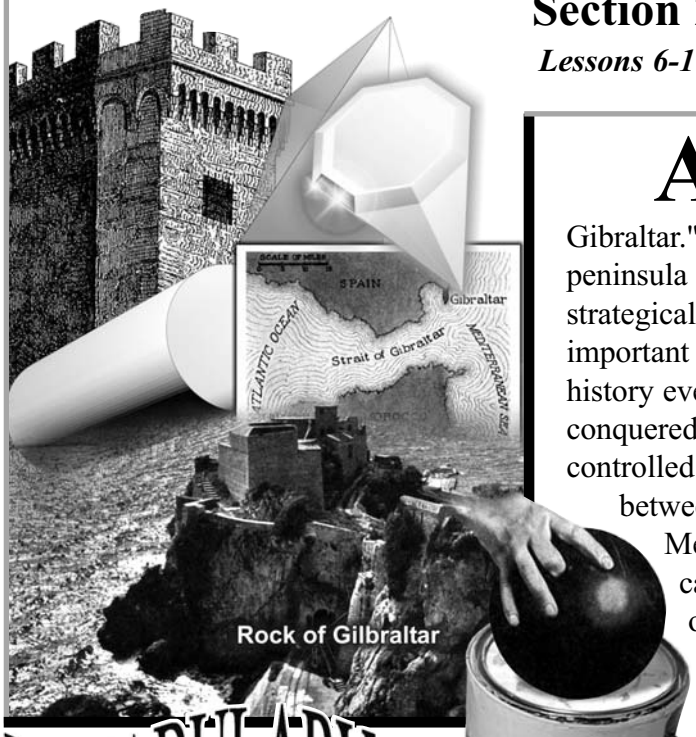
Geometry

Chapter 4

Section 2

Lessons 6-10





A common saying today refers to something being "***solid*** as the Rock of Gibraltar." Gibraltar is a British colony located on a peninsula in the Mediterranean Sea. The peninsula is strategically located in such a place that makes it important to surrounding countries. Throughout history every major power in the region has conquered and controlled Gibraltar because whoever controlled this peninsula controlled the passageway between the Atlantic Ocean and the Mediterranean Sea. Almost undefeatable castles and fortresses were built on the ridge of limestone and shale of the peninsula.

The Rock of Gibraltar is a large, overwhelming mountain that dominates the surrounding countryside. When people say something is "solid as the Rock of Gibraltar," they mean that it is unmovable and strong. A person's strength of character is also sometimes compared to a rock, which means he has the moral courage to remain steadfast during attacks of misfortune.

In mathematics, a *solid* is a geometric figure that has three dimensions and consists of all the points on the surface and in the interior. In this lesson, you will learn the different names for various solids and terms related to solids.

VOCABULARY

Solid: consists of all the points on the surface and all the interior points
Simple closed surface: consists of exactly one interior, contains no holes, and is hollow
Sphere: the set of all points equal distance from a given point or center; a ball
Face: a side or surface of a polygon or other object
Polyhedron: a simple closed surface composed of faces
Prism: a polyhedron consisting of two congruent parallel faces in the shape of a polygon, connected by parallelograms

Definition: Simple Closed Surface

A ***simple closed surface*** consists of exactly one interior, contains no holes, and is hollow.

A basketball is an example of a *simple closed surface*. The basketball has an interior, or inside, it contains no holes (or it would go flat), and it has a hollow area that is filled with air. A bowling ball would be an example of something that is not a simple closed surface. A bowling ball has holes in it for your fingers to grip. A bowling ball is not hollow.

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Definition: Sphere

A ***sphere*** is the set of all points equal distance from a given point or center.



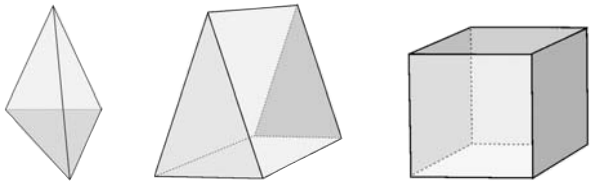
A basketball is an example of a *sphere*. Any point on the surface of the basketball is the same distance from the center as any other point on the surface of the basketball. A globe, the model of the earth, is also a sphere for the same reason.

Remember, a circle is the set of all points equal distance from a center, just as in a sphere. The difference between a circle and a sphere is that the circle is defined in two dimensions (a plane or flat surface) and a sphere is defined in three dimensions or space.

Definition: Solid

A solid is the set of all points on the surface and all the interior points.

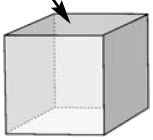




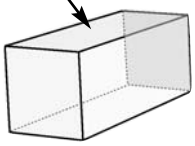
Definition: Face

A ***face*** is the side of a ***polyhedron***. A face is also a region in a plane that is in the shape of a polygon.

face of a cube

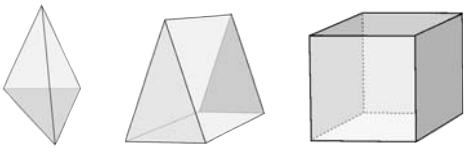


face of a square prism



Definition: Polyhedron

A polyhedron is a simple closed surface consisting of faces. A polyhedron is a solid shape with many faces (poly).



VOCABULARY

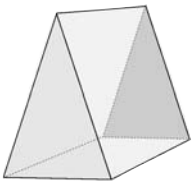
Base: the end face of a polyhedron
Side: a face of a polyhedron that is not a base
Cube: a polyhedron with six faces and all faces are squares
Cone: a geometric figure with a circular base and tapers to a point
Cylinder: a geometric figure consisting of two circular parallel faces
Pyramid: a geometric solid with a polygon base and triangular faces that connect at one point

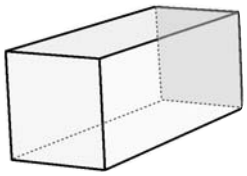
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Definition: Prism

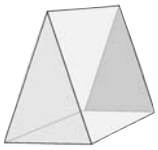
A ***prism*** is a polyhedron with two congruent (same) parallel faces in the shape of a polygon, connected by parallelograms.

Triangular prism Square prism





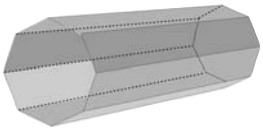
Prisms are named for the polygon that forms the two parallel bases. The end faces are called ***bases***. The other faces that are not bases are called ***sides***. For example if the two parallel bases of a prism are triangles, then the prism is a triangular prism. If the two parallel bases are rectangles, then the prism is a rectangular prism. An octagonal prism is a prism with octagons (eight sides) as parallel bases. A ***cube*** is a polyhedron with six sides that are all squares.



Triangular Prism



Rectangular Prism



Octagonal Prism

Definition: Cone

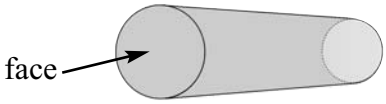
A ***cone*** is a geometric figure with a circular base and tapers to a point.



Examples of cones are ice cream cones and the tops of funnels.

Definition: Cylinder

A cylinder is a geometric figure that has two circular parallel faces.

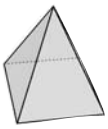


The piston of an automobile engine is an example of a cylinder. Paper towel and toilet paper rolls are in the shape of cylinders, as are soft drink cans.

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Definition: Pyramid

A ***pyramid*** is a geometric solid with a polygon base and triangular faces that connects at one point.



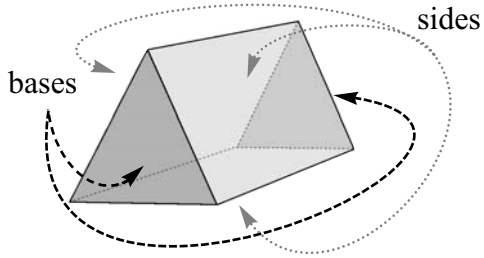
Rule

The number of faces on a polyhedron is obtained by adding the number of bases and the number of sides. The number of faces of the polyhedron is equal to the number of sides plus the bases.

To determine the number of faces on a polyhedron, first determine the number of sides on the base. Sometimes this requires some thought, but if you take your time you can think your way through the question.

Example Set 1

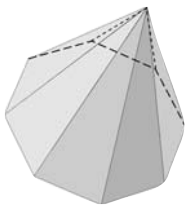
A *triangular prism* has triangle-shaped bases, which are connected by rectangles. Since the bases are in the shape of triangles, the prism will have three sides. A prism has two bases, therefore, two bases plus three sides add together to get five faces on a triangular prism.



3 sides + 2 bases = 5 faces

Example Set 2

To determine the number of faces on an octagonal pyramid, you must think of what a pyramid looks like and use the same thought process as with the prism. The base of an octagonal pyramid is in the shape of an octagon (eight sides). A pyramid has only one base. Since the base has eight sides, then the pyramid will have eight sides. One base and eight sides add together to obtain nine faces on an octagonal pyramid.



1 base + 8 sides = 9 faces

The number of faces on any polyhedron is obtained in the same manner.

Apply what you have learned about the solid shapes in geometry by working the activities for this lesson. In the next lesson you will learn how to find surface areas of geometric solids.

LIFE PRINCIPLE: **"A person of moral courage is steadfast and unmovable during temptation."**

—Hebrew Principle



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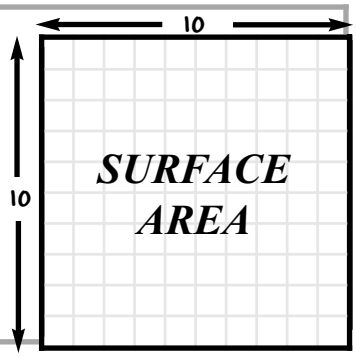
Lesson 9

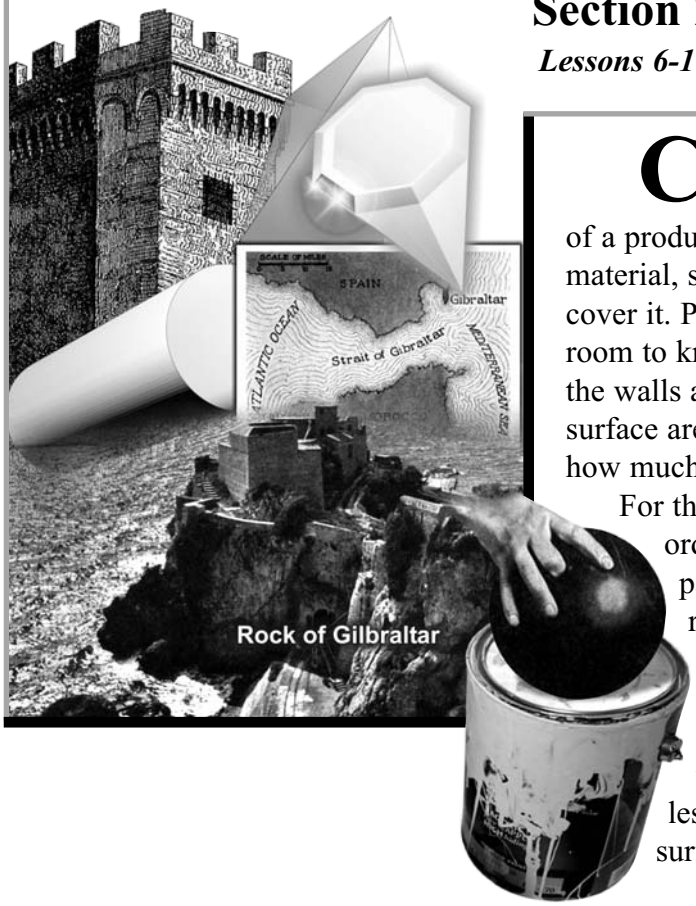
Geometry

Chapter 4

Section 2

Lessons 6-10





Companies that manufacture products sometimes need to know the surface area of a product in order to determine the amount of material, such as cloth or metal, that it will take to cover it. Painters need to know the surface area of a room to know how much paint is needed to cover the walls and ceiling. Builders need to know the surface area of the outside of a building to know how much siding is needed to cover the building. For the inside of the building, the builder must order the proper amount of sheet rock or paneling to cover the surface area of all the rooms.

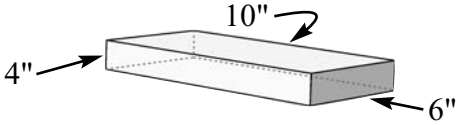
Determining ***surface areas of solid*** objects involves knowing how to find the areas of various polygons and then adding these areas together. In this lesson you will learn how to determine the surface areas of various solids.

Finding the surface area of a rectangular prism requires calculating the area of six rectangles (the six faces of the rectangular prism). Follow the calculations as shown in **Example Set 1.**

Example Set 1

A rectangular prism is 4" high, 6" wide, and 10" deep. Find the surface area of the rectangular prism.

Step 1 Draw a picture of the prism from the information given.



VOCABULARY

Surface area of a solid: the sum of all the areas of the faces of a polyhedron; the area of the surface of a sphere

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Step 2 Notice that six rectangles make up the faces of the rectangular prism. The two bases have the dimensions, 4" × 6". Two sides have dimensions, 6" × 10", and two sides have dimensions, 4" × 10".

Step 3 The area of each base and side must be found.
Area Base 1 = 4 × 6 = 24
Area Base 2 = 4 × 6 = 24
Area Side 1 = 6 × 10 = 60
Area Side 2 = 6 × 10 = 60
Area Side 3 = 4 × 10 = 40
Area Side 4 = 4 × 10 = 40

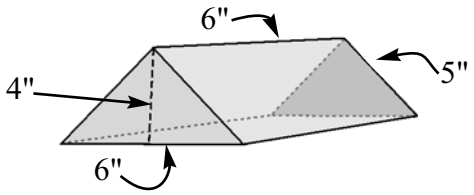
Step 4 The six areas calculated in Step 3 add together to obtain the total surface area of the prism. 24 + 24 + 60 + 60 + 40 + 40 = 248 sq in.

Finding the surface area of triangular prisms is the same as with rectangular prisms. Follow the steps in **Example Set 2** to find the surface area of a triangular prism.

Example Set 2

Find the surface area of a triangular prism of height 4", width 6", and depth 6".

Step 1 Draw a picture of the triangular prism.



Step 2 Notice a triangular prism has two bases and three sides (the number of sides of the base). Therefore, five preliminary area calculations must be done before finding the total surface area of the triangular prism. The triangular bases have a base (b) of 6" and a height (h) of 4". The formula for calculating the area of a triangle is $A = \frac{1}{2}bh$. Two sides are rectangles of width 5" and length 6". One side is a rectangle of width 6" and length 6". The area of these rectangles will be calculated using the formula for the area of a rectangle, $A = l \times w$.



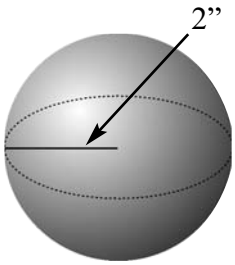
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Step 3 The areas of the five faces are calculated.
Area of base 1 = $\frac{1}{2}$ (6)(4) = $\frac{1}{2}$ (24) = 12
Area of base 2 = $\frac{1}{2}$ (6)(4) = $\frac{1}{2}$ (24) = 12
Area of side 1 = 6(6) = 36
Area of side 2 = 5(6) = 30
Area of side 3 = 5(6) = 30

Step 4 Add the areas of the five faces calculated in Step 3 together to obtain the surface area of the triangular prism. 12 + 12 + 36 + 30 + 30 = 120 sq in.

Calculating the surface area of a sphere requires a special formula since a sphere has no faces that can have the areas calculated and added together. The formula for calculating the area of a sphere is $A = 4\pi r^2$, where r is the radius of the sphere. Look at **Example Set 3** to see how to use this formula to calculate the surface area of a sphere.

Example Set 3



A sphere has a radius of 2 inches. Calculate the surface areas of the sphere using $\frac{22}{7}$ for the value of π. Use the formula, $A = 4\pi r^2$, and substitute $r = 2$ into the formula to obtain the following equation.

$$A = 4\pi(2)^2$$

$$2^2 = 2 \times 2 = 4$$

Substitute *4* for 2^2 and $\frac{22}{7}$ for π, to obtain the equation: $A = 4(\frac{22}{7})(4)$. Perform the calculations as follows to obtain the surface area of a sphere of radius 2 inches:

$$A = \frac{88}{7}(4) = \frac{352}{7} = 50\frac{2}{7} \text{ sq in.}$$

Apply what you have learned about finding the surface area of solid shapes in geometry by working the activities for this lesson. In the next lesson you will learn how to find the surface areas of other geometric solids.

LIFE PRINCIPLE



"You can easily calculate a person's responses when he is known for honesty."
—Kentucky Axiom

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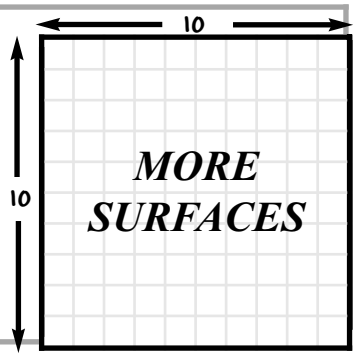
Lesson 10

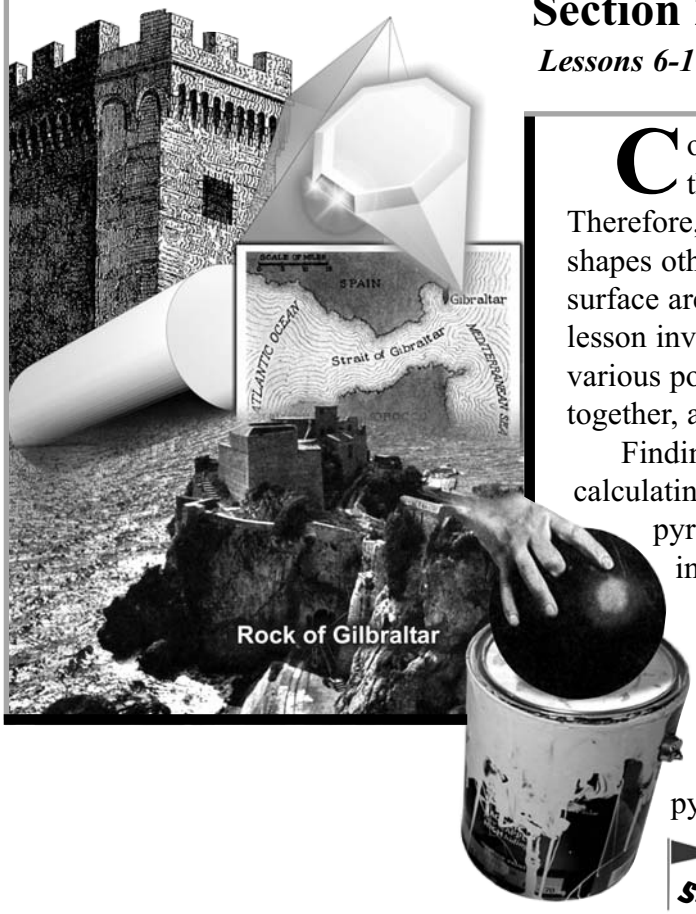
Geometry

Chapter 4

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Companies that manufacture products make those products of various shapes. Therefore, they need to know the surface area of shapes other than prisms and spheres. Determining surface areas of the solid objects discussed in this lesson involves knowing how to find the areas of various polygons and then adding these areas together, as in the previous lesson.

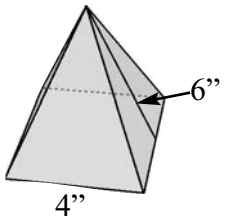
Finding the surface area of a pyramid requires calculating the area of the base and the sides of the pyramid. Follow these calculations as shown in **Example Set 1**.

Example Set 1

A square pyramid has a base with sides of 4". The measure of the ***slant height*** is 6". Find the surface area of the square pyramid.

Step 1

Draw a picture of the pyramid from the information given.



VOCABULARY

Slant height: the altitude of the triangular face of a pyramid

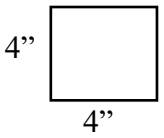
Step 2

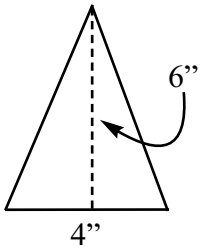
Notice that four triangles (sides) and a square (base) make up the faces of the square pyramid. The base has the dimensions, 4" × 4". The four triangular sides have dimensions of base, b = 4" and altitude (height, h) = 6".

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Step 3 The area of the base and each side must be found. The formula for the area of a triangle is $A = \frac{1}{2}bh$, where b is the measure of the base and h is the measure of the height.

Area Base = 4 × 4 = 16





Area Side 1 = $\frac{1}{2}$ × 4 × 6 = 2 × 6 = 12
Area Side 2 = $\frac{1}{2}$ × 4 × 6 = 2 × 6 = 12
Area Side 3 = $\frac{1}{2}$ × 4 × 6 = 2 × 6 = 12
Area Side 4 = $\frac{1}{2}$ × 4 × 6 = 2 × 6 = 12

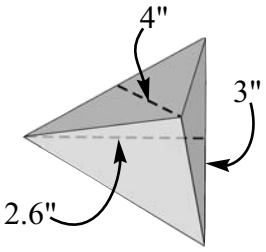
Step 4 The five areas calculated in Step 3 are added together to obtain the total surface area of the pyramid. 16 + 12 + 12 + 12 + 12 = 64 sq in.

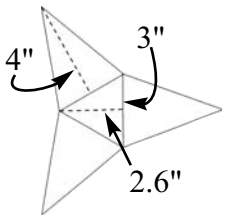
Finding the surface area of triangular pyramids is the same as with rectangular or square pyramids. Follow the steps in **Example Set 2** to find the surface area of a triangular pyramid.

Example Set 2

Find the surface area of a triangular pyramid of slant height 4" and a triangular base of altitude, 2.6" and base of 3".

Step 1 Draw a picture of the triangular pyramid.





Step 2 Notice a triangular pyramid has one base and three sides (the number of sides of the base). Therefore, four preliminary area calculations must be done before finding the total surface area of the triangular pyramid. The triangular base is an equilateral triangle with each side of 3" and a height (altitude) of 2.5". The sides are also triangles of base 3" and height 4". The area of all these triangles will be calculated using the formula for the area of a triangle, $A = \frac{1}{2}bh$.

Step 3 The areas of the four faces are calculated.
Area of the base = $\frac{1}{2}$(2.6)(3) = $\frac{1}{2}$(7.5) = 3.9
Area of side 1 = $\frac{1}{2}$(3)(4) = $\frac{1}{2}$(12) = 6
Area of side 2 = $\frac{1}{2}$(3)(4) = $\frac{1}{2}$(12) = 6
Area of side 3 = $\frac{1}{2}$(3)(4) = $\frac{1}{2}$(12) = 6



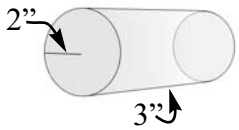
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Step 4 Add the areas of the four faces calculated in Step 3 together to obtain the surface area of the triangular pyramid. 3.9 + 6 + 6 + 6 = 21.9 sq in.

Calculating the surface area of a cylinder requires understanding which polygons are used to form a cylinder. A paper towel roll is an example of a cylindrical shape. Think of what a paper towel roll looks like. Each end is a circle. If the side of the paper towel roll were cut lengthwise along a straight line and then the roll were laid out flat, the polygon formed would look like a rectangle. Look at **Example Set 3** to see how to use this information to calculate the surface area of a cylinder.

Example Set 3

A cylinder has a radius of 2 inches and a height of 3 inches. Calculate the surface area of the cylinder using $^{22}/_{7}$ for the value of π.

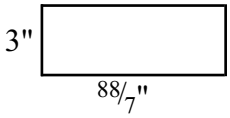


Step 1 Calculate the area of the ends. Each end is a circle and the area must be calculated. The area of a circle is calculated using the formula $A = \pi r^2$. Substitute the value of the radius into the formula for the area of a circle to find the area of each end of the cylinder. This calculation will be used twice in the final calculation of surface area of the cylinder because the cylinder has two circular ends.

$A = \pi(2)^2 = \pi(4) = {}^{22}/_{7}(4) = {}^{88}/_{7}$ sq in.

Step 2 The dimensions of the rectangle that form the side of the cylinder are the width (= the height of the cylinder) and the length (= the circumference of the circle that defines the end of the cylinder). The height of the cylinder given in the problem is 3". The circumference of the ends must be calculated using the formula for circumference of a circle, $C = \pi d$, where d is the diameter of the circle. Since the radius of the end of the cylinder is 2", the diameter is two times the radius or 4". The calculation of the length of the rectangle forming the side of the cylinder is then:

$C = \pi d = \pi(4) = {}^{22}/_{7}(4) = {}^{88}/_{7}$ inches



Step 3 The measurements of the rectangle forming the cylinder is shown in Step 2. The area of the rectangle is calculated as: $A = l \times w = 3 \times {}^{88}/_{7} = {}^{264}/_{7}$ sq in.

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Step 4 The three areas calculated are added together to obtain the surface area of the cylinder.

$$A = \overbrace{\frac{88}{7} + \frac{88}{7}}^{\text{ends}} + \overbrace{\frac{264}{7}}^{\text{rectangle}} = \frac{176}{7} + \frac{264}{7} = \frac{440}{7} = 62\frac{6}{7} \text{ sq in.}$$

Apply what you have learned about the finding the surface area of solid shapes in geometry by working the activities for this lesson. In the next lesson you will learn how to find volumes of geometric solids.

LIFE PRINCIPLE



"A person is known by his actions."

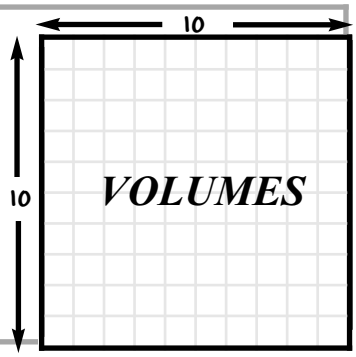
—Hebrew Proverb

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Geometry

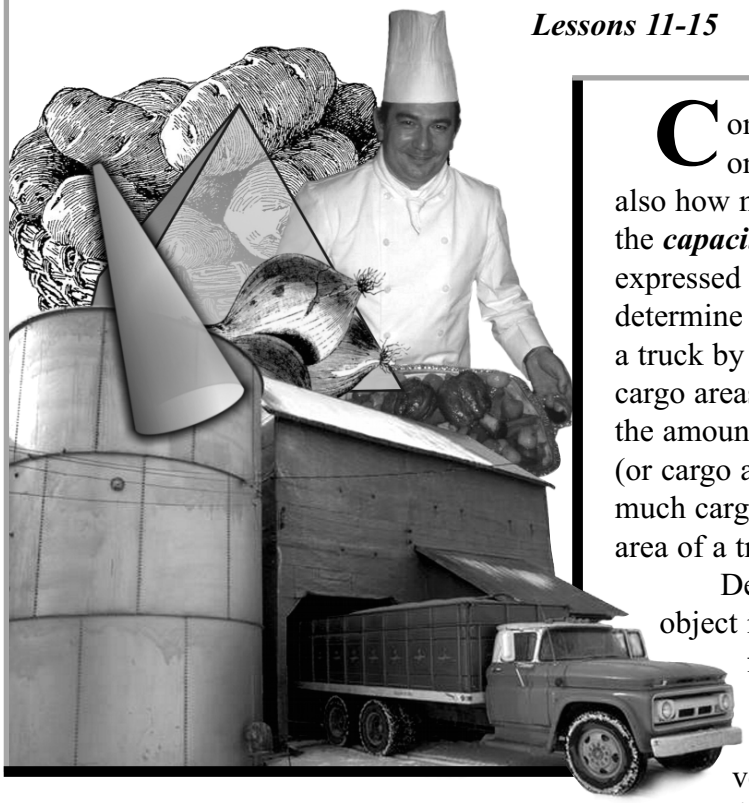
Lesson 11

Chapter 4



Section 3

Lessons 11-15



Companies that manufacture products not only need to know surface areas, but also how much space something occupies, and the ***capacity*** of a container. Often, volumes are expressed in *cubic units*. Trucking companies determine the *space* of trailers or cargo areas of a truck by calculating the volume of trailers (or cargo areas). Shipping companies also calculate the amount of space *objects* occupy in trailers (or cargo area). Then the companies know how much cargo can be hauled in trailers (or cargo area of a truck).

Determining the ***volume of a solid*** object involves knowing how to use formulas for calculating the volume of each type of solid. In this lesson you will learn how to determine the volume of various solids.

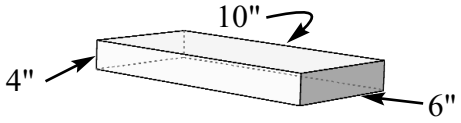
Finding the volume of a rectangular prism requires using the formula, $V = l \times w \times h$, where l is the length, w is the width, and h is the height. Follow the calculations as shown in **Example Set 1** to find the volume of a rectangular prism.

Example Set 1

A rectangular prism is 4" high, 6" wide, and 10" deep. Find the volume of the rectangular prism.

Step 1

Draw a picture of the prism from the information given.



VOCABULARY

Capacity: how much a shape can hold

Volume of a solid: how much space the solid takes up

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Step 2 Substitute the dimensions of the rectangular prism given into the formula for the volume of a rectangular prism as follows:
$h = 4"$, $l = 10"$, and $w = 6"$

$$V = l \times w \times h = 10 \times 6 \times 4 = 240 \text{ cubic inches}$$

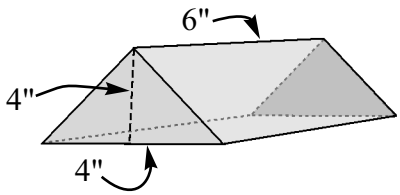
Therefore the volume of a rectangular prism 4" high, 6" wide, and 10" deep is 240 cubic inches.

Finding the volume of a triangular prism is similar to calculating a rectangular prism. However, since the bases are triangles instead of rectangles the formula for calculating the volume of a triangular prism also looks similar to the formula for calculating the area of a triangle. Follow the steps in **Example Set 2** to find the volume of a triangular prism.

Example Set 2

Find the volume of a triangular prism of height 4", width 4", and depth 6".

Step 1 Draw a picture of the triangular prism.





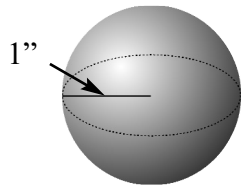
Step 2 The formula for the volume of a triangular prism is $V = \frac{1}{2}bhl$, where b is the measure of the base, h is the height, and l is the length. Substitute the values of b, h, and l into the formula for the volume of a triangular prism to obtain the following equation. $V = \frac{1}{2}bhl = \frac{1}{2}(4)(4)(6) = (2)(4)(6) = 8(6) =$ 48 cubic inches. The volume of a triangular prism of width 4", height 4", and length 6" is 48 cubic inches. Notice that the words *base* and *width* can be used interchangeably; thus, the formula for the volume of a triangular prism can also be stated as $V = \frac{1}{2}\,lwh$.

Calculating the volume of a sphere requires another special formula since a sphere has no dimensions that can be multiplied together. The formula for calculating the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. Look at **Example Set 3** to see how to use this formula to calculate the volume of a sphere.

Example Set 3

A sphere has a radius of 1 inch. Calculate the volume of the sphere using $\frac{22}{7}$ for the value of π.

Step 1 Draw a picture of the sphere.



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Step 2 Use the formula, $V = \frac{4}{3}\pi r^3$, and substitute $r = 1$ into the formula to obtain the following equation and follow the calculations to find the volume of the sphere.
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi(1)(1)(1) = \frac{4}{3}(\frac{22}{7}) = \frac{88}{21} = 4\frac{4}{21}$ cubic inches

Apply what you have learned about finding the volume of solid shapes in geometry by working the activities for this lesson. In the next lesson you will learn how to find volumes of other geometric solids.

LIFE PRINCIPLE



"Your speech and your actions speak volumes about your character."

—Anonymous

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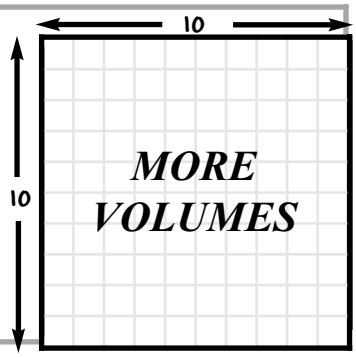


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Geometry

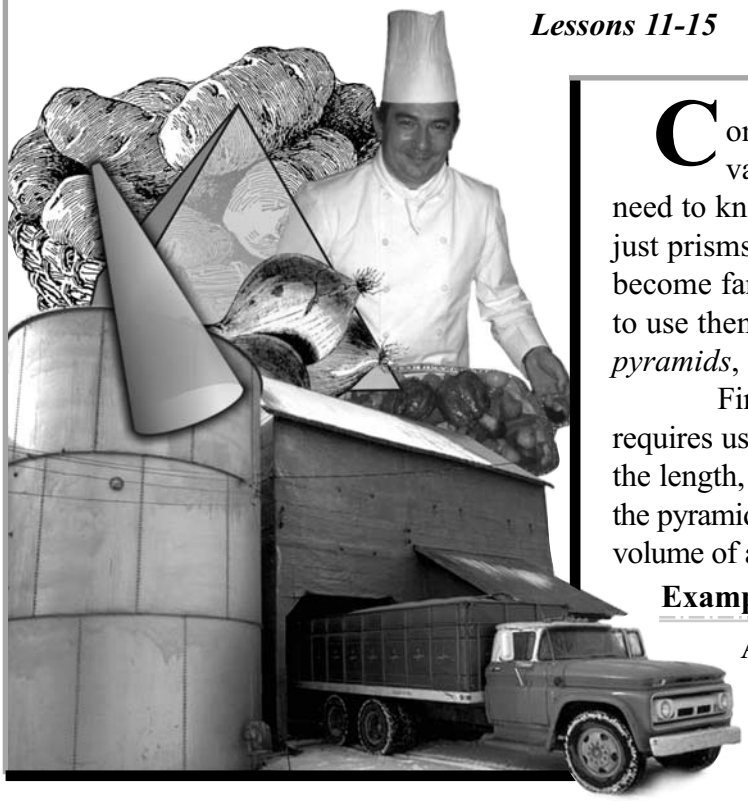
Lesson 12

Chapter 4



Section 3

Lessons 11-15



Companies manufacture products of various shapes. Therefore, producers need to know the volume of shapes other than just prisms and spheres. In this lesson you will become familiar with various formulas and how to use them in calculating the volumes of *pyramids*, *cylinders* and *cones*.

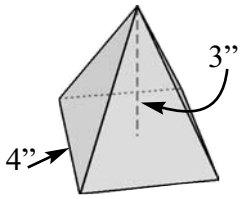
Finding the volume of a pyramid requires using the formula $V = (1/3)lwh$, where l = the length, w = the width, and h = the height of the pyramid. Follow the calculations for finding the volume of a pyramid as shown in **Example Set 1**.

Example Set 1

A square pyramid has a base with sides of 4". The measure of the ***altitude*** is 3". Find the volume of the square pyramid.

Step 1

Draw a picture of the pyramid from the information given.



Step 2

Use the formula for the volume of a pyramid, $V = (1/3)lwh$, and substitute the values for l, w and h given in the problem into the formula. $l = 4"$, $w = 4"$ and $h = 3"$. $V = (1/3)lwh = (1/3)(4)(4)(3) = (1/3)(16)(3) = (1/3)(48) = 48/3 =$ 16 cubic inches. The volume of a pyramid with a 4" × 4" base and an altitude of 3" is 16 cubic inches.

VOCABULARY

Altitude: the perpendicular height of an object. (The words height and altitude are interchangeable.)

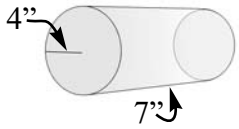
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The formula for calculating the volume of a cylinder is $V = \pi r^2 h$, where r is the radius of an end and h is the height of the cylinder. Look at **Example Set 2** to see how this formula is used to calculate the volume of the cylinder described.

Example Set 2

Find the volume of a cylinder with a height of 7" and a radius of 4".

Step 1 Draw a picture of the cylinder.



Step 2 Place the values for height, h = 7" and r = 4" into the formula for the volume of a cylinder as shown below and perform the calculations to obtain the volume of the cylinder.

$V = \pi r^2 h = {}^{22}/_{7}(4)^2(7) = {}^{22}/_{7}(16)(7) = (22)(16) = 352$ cubic inches

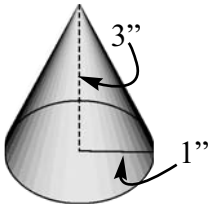


Calculating the volume of a cone requires a formula similar to that of a pyramid, but with π as a factor (multiplier) in the formula, since a cone has a circular shape for the base. The formula for the volume of a cone is $V = {}^{1}/_{3}\pi r^2 h$. Notice that the formula for calculating the volume of a cone is also almost identical to that of the formula for calculating the volume of a cylinder except for the factor, ${}^{1}/_{3}$. If a cylinder and cone have bases of the same radius, and a cylinder and cone have the same height, then the cone is one-third the volume of the cylinder. To prove this, fill a cup in the shape of a cone with water. Then pour the water into a cup in the shape of a cylinder that has the same radius as the cone and the same height. You will find that three cone-shaped cups of water equal the cylinder-shaped cup full of water.
Look at **Example Set 3** to see how to use the formula for calculating the volume of a cone.

Example Set 3

A cone has a radius of 1 inch and a height of 3 inches. Calculate the volume of the cone using ${}^{22}/_{7}$ for the value of π.

Step 1 Draw a picture of the cone described in the problem so you can visualize the problem.



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Step 2 Place the values of the radius and height into the formula for the volume of a cone. Follow the calculations as shown to find the volume of the cone.
$V = {}^{1}/_{3}\pi r^2 h = {}^{1}/_{3}({}^{22}/_{7})(1)^2(3) = ({}^{22}/_{21})(1)(3) = ({}^{22}/_{21})(3) = {}^{22}/_{7} = 3{}^{1}/_{7}$ cubic inches

Apply what you have learned about finding the volume of solid shapes in geometry by working the activities for this lesson.

LIFE PRINCIPLE



"The things with which you fill your life help determine who you are and what you will become."
—Anonymous

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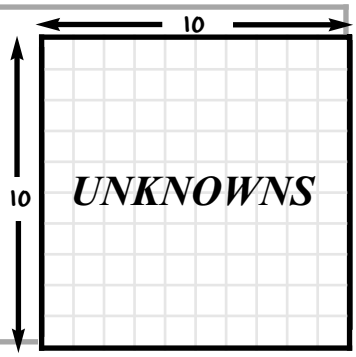
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Lesson 13

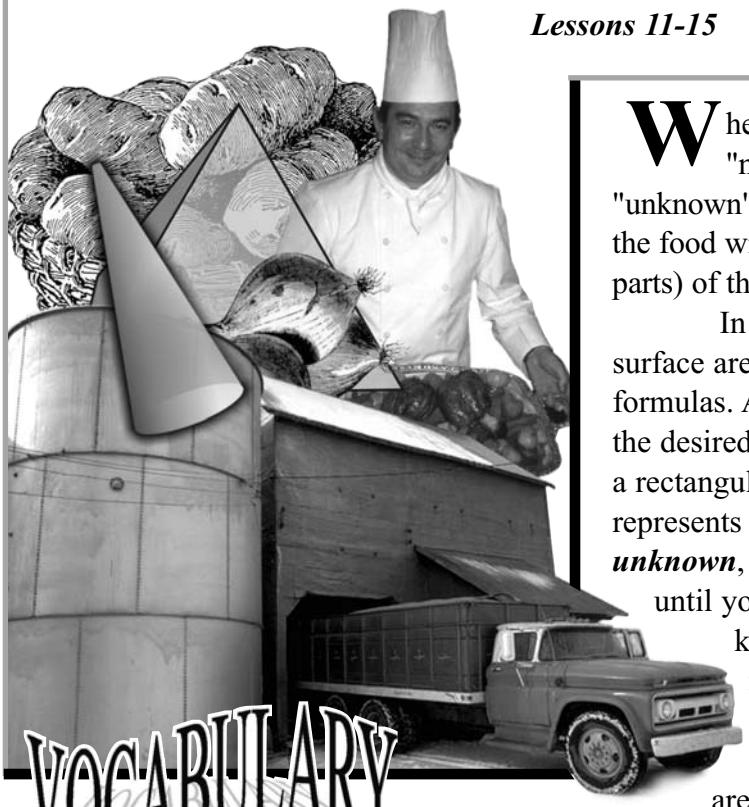
Algebra

Chapter 4



Section 3

Lessons 11-15



When a cook follows a recipe to bake a "new" item, the finished product is the "unknown" because the cook does not know how the food will turn out. The ingredients (individual parts) of the recipe are the *known* quantities.

In the previous lessons, you calculated surface area and volumes of solids using formulas. A formula is a "recipe" to determine the desired result. The formula for the volume of a rectangular prism is $V = l \times w \times h$. The V represents the volume and the V is the ***unknown***, because you do not know what V is until you calculate it. The l, w and h are known quantities, just like the ingredients of a recipe.

Often when working with formulas, the unknown quantity you are calculating is already *isolated* (by itself) on one side of the equals sign. However, sometimes the equation must be *manipulated* (changed around) to obtain an equation with the unknown isolated on one side of the equals sign. Sometimes the quantity you are asked to find is combined with other numbers by multiplication, division, addition or subtraction.

When working problems in mathematics the answer is not always as straightforward as substituting values for letters and performing the calculations. The unknown (letter) must be isolated or set apart on one side of the equal sign with all the other terms of the equation on the other side of the equal sign.

In this lesson you will see how the value of an unknown can be obtained through following a series of steps and rules. Look at the following rules and then follow the steps in the examples to see how the rules are applied to equations to ***solve an equation*** for an unknown quantity.

VOCABULARY

Unknown: the unknown value in an equation; usually, but not always, denoted by a lower case letter such as the letter x or y.
Solve an equation: perform various operations on an equation to obtain a solution
Solution: the answer to an equation containing an unknown; the value of the unknown set: a group of related objects
Set symbols: { }
Solution set: a set of all the solutions to a problem

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RULE 1



If an operation is performed on one side of an equation, then the exact operation must be performed on the other side of the equation. (Properties of Equality)



- $5 = 5 \rightarrow 5 + 2 = 5 + 2 \rightarrow 7 = 7$. When 2 is added to both sides of the equation, both sides are still equal.
- $23 = 23 \rightarrow 23 \times 2 = 23 \times 2 \rightarrow 46 = 46$. In this example, both sides of the equation are multiplied by 2. The result equation still represents an equality.

RULE 2



To move a term to the other side of the equal sign, the opposite operation is performed.

- $5 + 3 = 8$ (isolate the 5) $\rightarrow 5 + 3 - 3 = 8 - 3$ (subtract 3 from both sides) $\rightarrow 5 = 8 - 3$
- $15 - 4 = 11$(isolate the 15) $\rightarrow 15 - 4 + 4 = 11 + 4$ (add 4 to both sides) $\rightarrow 15 = 11 + 4$

RULE 3



When isolating the unknown using **Rule 2**, all additions and subtractions are performed before multiplications or divisions.

- $2(3) + 5 = 11$ (isolate the 3) $\rightarrow 2(3) + 5 - 5 = 11 - 5$ (subtract 5 from both sides) $\rightarrow 2(3) = 6 \rightarrow 2(3) \div 2 = 6 \div 2$ (divide both sides by 2) $\rightarrow 3 = 6 \div 2$.
- $4(5) - 8 = 12$ (isolate the 5) $\rightarrow 4(5) - 8 + 8 = 12 + 8$ (add 8 to both sides) $\rightarrow 4(5) = 20 \rightarrow 4(5) \div 4 = 20 \div 4$ (divide both sides by 4) $\rightarrow 5 = 20 \div 4$

Example Set 1

Find the ***solution*** to $x - 5 = 6$.

Step 1 Look at the equation to see that "x" is the unknown quantity.

Step 2 Looking at **Rule 1**, we know that "x" must be isolated on one side of the equal sign so that a solution is obtained. <u>If possible, the standard form (the form to write the answer) of an equation containing an unknown is to isolate the unknown to the left hand side of the equation (lhs).</u>

Step 3 In the problem, $x - 5 = 6$, the operation performed on the unknown, x, is subtraction ($x - 5$). Rule 2 states that to move the 5 to the right hand side (rhs) of the equation, the opposite operation must be performed. Therefore, 5 must be added to both sides of the equation to isolate the x.

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Step 4 The equation resulting from Step 3 is: $x - 5 + 5 = 6 + 5$

Step 5 The calculations in Step 4 are performed. ($5 - 5 = 0$ and $6 + 5 = 11$) to obtain the following equation. $x = 11$

Since x is isolated on the lhs, the solution to the original equation is obtained and the solution is placed in ***set symbols***, {11}, to signify the ***solution set*** of the problem $x - 5 = 6$.

The operations of addition or subtraction are not the only operations that are performed on unknowns. The operations of multiplication and division are often involved in isolating the unknown as is shown in **Example Set 2**.

Example Set 2

Find the solution to the following problem.

$\frac{x}{5} = 5$

Step 1 In this problem, five divides the unknown. (Remember, fractions are another method of signifying division.)

Step 2 Following the procedure set forth in Rule 2, each side of the equation must be multiplied by 5. This yields the equation: $\frac{x}{5} \bullet 5 = 5 \bullet 5$

Step 3 Performing the operations in Step 3 ($5 \div 5 = 1$; $5 \bullet 5 = 25$) yields the following equations. $x\,(1) = 25 \rightarrow x = 25$

Step 4 As obtained in Step 3, 25 is the solution to the equation $\frac{x}{5} = 5$ and 25 is placed in a solution set. {25}

Many problems require multiple operations to obtain the solution. Look at **Example Set 3** to see how this is accomplished.

Example Set 3

Find the solution to $6x + 5 = 11$.

Step 1 The unknown must be isolated first by subtracting 5 from each side of the equation (**Rule 3** and **Rule 2**).

$6x + 5 - 5 = 11 - 5$

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Step 2 Performing the calculations in Step 1, (5 - 5 = 0 and 11 - 5 = 6) yields the following equation.

$6x = 6$

Step 3 The new equation obtained in Step 2 is now solved using Rule 2. Six must divide both sides of the equation (Rule 2).

$\frac{6x}{6} = \frac{6}{6}$

Step 4 The calculations in Step 3, ($6 \div 6 = 1$) yields the solution to the original problem $6x + 5 = 11$.

$x = 1$

Step 5 The solution, 1, is placed in set symbols to signify the solution set to the equation $6x + 5 = 11$.

$\{1\}$

Apply what you have learned about solving equations containing one unknown by working the activities for this lesson.

LIFE PRINCIPLE



"Do unto others as you would have them do unto you."

—The Golden Rule

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x+3=7

Lesson 14

Algebra

Chapter 4

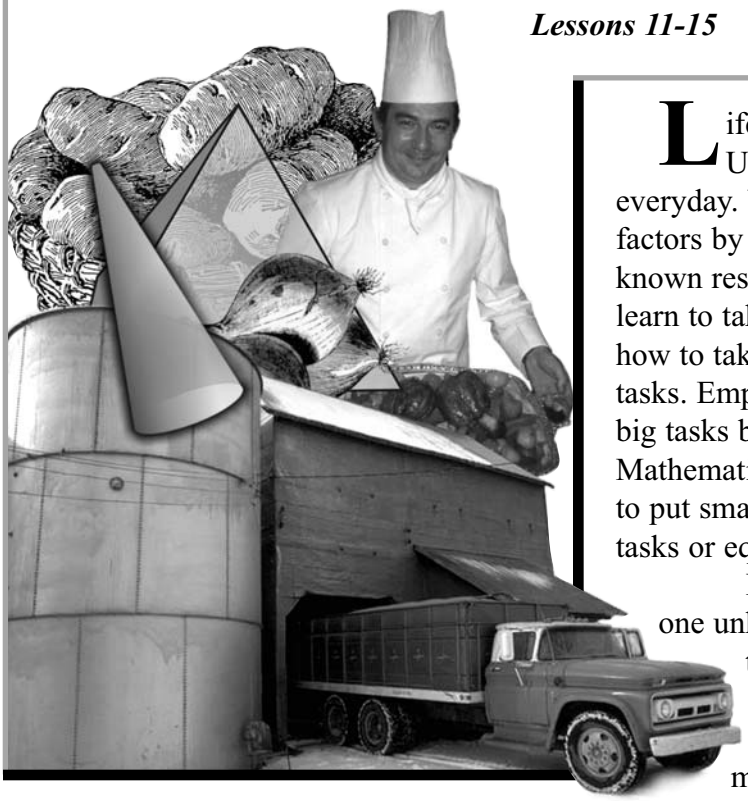
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10

MULTIPLE UNKNOWNS

Section 3

Lessons 11-15





Life is not always ***predictable***. Unknown factors affect us everyday. We can prepare for unknown factors by learning to take care of known responsibilities. People who learn to take care of simple responsibilities learn how to take care of more complex or difficult tasks. Employment often requires us to solve big tasks by taking care of smaller tasks first. Mathematics is a means by which we learn how to put small tasks in ***sequence*** to solve bigger tasks or equations.

Many equations contain more than one unknown and finding the solutions to these equations is more involved. Formulas in previous lessons for finding various quantities contain more than one unknown. Another formula common in mathematics is the time and distance formula stated below.

Time and Distance Formula

Distance traveled is calculated using the formula, distance = rate times time, ($d = r \bullet t$), where d = distance traveled, r = rate of speed, and t = time.

Example Set 1

a) Calculate how many miles you would have traveled if you drove for two hours at 55 mph. Place the known values from the problem, r = 55 mph and t = 2 hrs, into the formula for distance, $d = r \bullet t$.

$d = r \bullet t$ = 55 mph (2 hrs) = 110 miles

At a rate of 55 mph in two hours you would have driven 110 miles.

VOCABULARY

Predictable: able to be predicted or forcasted
Sequence: the following of one thing after another in an orderly way

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b) Calculate how many miles a spaceship will travel in 4 days at a speed of 18,000 mph.

Step 1 The time units in 4 days does not match the time units in the rate, 18,000 mph (miles per hour). One unit is *days* and the other unit is *hours*, therefore a conversion must be completed to make the units the same. One day equals twenty-four hours, therefore, 4 days • 24 hours = 96 hours.

Step 2 Place the known values from the problem into the formula for distance, $d = r \bullet t$.

$$d = r \bullet t = 18{,}000 \text{ mph} \bullet 96 \text{ hours} = 1{,}728{,}000 \text{ miles.}$$

A space ship traveling at a speed of 18,000 mph for 4 days will travel 1,728,000 miles.

The *time and distance formula* and the other formulas studied earlier will provide equations containing more than one unknown for use in this lesson. When solving equations with more than one unknown you must follow the same rules as if the unknowns were numbers. Refer to the rules for solving equations from the previous lesson as you look at the following examples and follow the steps to understand how to solve these equations.

In **Example Set 2** the formula for the perimeter of a rectangle is used to show how to move unknowns from one side of the equation to another.

Example Set 2

Solve the following formula for l.

$P = 2w + 2l$

Step 1 Look at the equation to see that "l" is the unknown quantity that is required to be isolated. In this problem leaving the l on the rhs will be easier than moving it to the lhs.

Step 2 The equation is $P = 2w + 2l$. The first step to isolating the l is to subtract $2w$ from each side of the equation as shown below. (Remember the rules from the previous lesson.)

$$P - 2w = 2w + 2l - 2w$$

Step 3 The resulting equation obtained from the operations performed in Step 2 is $P - 2w = 2l$. 2 multiplies l in this new equation, so to isolate the l, you must divide both sides of the equation by 2 as shown below.

Step 4 When the fraction, $^{2l}/_{2}$ is reduced, the equation resulting from Step 3 is: $^{(P-2w)}/_{2} = l$. The equation $P = 2w + 2l$ is now solved for l.

Sometimes we need to solve other formulas, such as the formula for the volume of a triangular prism, for a specific unknown or letter. **Example Set 3** shows how to solve the formula for the volume of a triangular prism for a specific unknown as was done in **Example Set 2** for the perimeter of a rectangle.

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Example Set 3

Solve the following formula for h.

$V = {}^1/_2\, bhl$

Step 1 All the operations involved are multiplications except division by 2 (multiplying by $^1/_2$ is the same as dividing by 2). Therefore, you must begin by dividing both sides of the formula by b and l as shown below.

$$V = \frac{1}{2}bhl \rightarrow \frac{V}{bl} = \frac{1\cancel{b}h\cancel{l}}{2\cancel{b}\cancel{l}} = \frac{h}{2}$$

Step 2 For ease of calculation the h will remain on the rhs of the equation as in the previous example. You must multiply both sides of the equation by 2 as shown below to completely isolate h on the rhs.

$$\frac{V}{bl} \bullet 2 = \frac{h}{2} \bullet 2$$

Step 3 Performing the operations in Step 2 ($2 \div 2 = 1$) yields the following equation with h isolated completely on the rhs.

$$\frac{2V}{bl} = h$$

Apply what you have learned about solving equations containing more than one unknown by working the activities for this lesson.

Life Principle



"Sometimes the best formula for facing the unknown is faith and truth."

—Elizabeth Chavez

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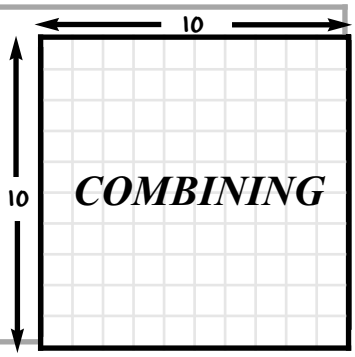
Basic Math Skills



Lesson 15

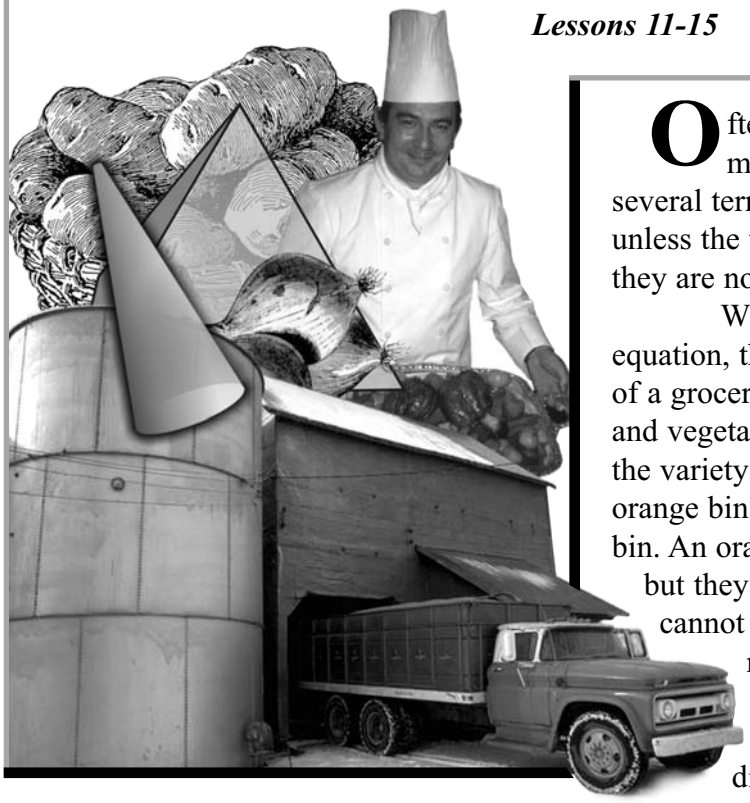
Algebra

Chapter 4



Section 3

Lessons 11-15



Often when solving problems in mathematics, an equation will contain several terms that look very similar. However, unless the terms have exactly the same unknown they are not considered ***like terms***.

When combining *like terms* in an equation, think of the fruit and vegetable section of a grocery store. The grocer separates the fruits and vegetables into different bins according to the variety of produce. The apples are not in the orange bin and the potatoes are not in the onion bin. An orange and a tangerine look very similar, but they have some differences that mean they cannot be combined in the same fruit bin. In mathematics, x and x^2 look very much alike but the exponents (^{1} and ^{2}) are different which makes the terms different.

Look at **Example Set 1** to become familiar with distinguishing between *like* and ***unlike terms***.

Example Set 1

Terms	Like or unlike	Reason
x, y	unlike	not the same letter(x and y)
$3x$, $5x$, $34x$	like	contain the same unknown, x
$2x^2$, $2x^3$	unlike	contain different unknowns because of the exponents
4, $6y$, $3y$	unlike	numbers alone are different terms than numbers with letters
$45w$, $5w$, $51w$, $3w$	like	contain the same unknown, w

VOCABULARY

Like terms: terms that contain the same unknowns (letters)
Unlike terms: terms that do not contain the same unknown (letter)

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In this lesson you will see how terms can be combined by following a simple rule. Look at the following rule and then follow the steps in the examples to see how the rule is applied to equations to solve for an unknown.

RULE

In an equation containing more than one term, only like terms can be added or subtracted before solving for the unknown.



Example Set 2

Find the solution to $2x - 5 = 6 - 9x$

Step 1 Look at the equation to see that "x" is the unknown quantity and there is an unknown (x) on both sides of the equals sign ($2x$ and $9x$).

Step 2 Looking at the Rule, we know that "x" must be isolated on one side of the equals sign so that a solution is obtained and therefore the like terms ($2x$ and $9x$) must be combined to accomplish this.

Step 3 In the problem, $2x - 5 = 6 - 9x$, the operation performed on the term, $2x - 5$, is subtraction ($2x - 5$). To move the 5 to the right hand side (rhs) of the equation, the opposite operation must be performed. Therefore, 5 must be added to both sides of the equation to isolate the $2x$.

Step 4 The equation resulting from Step 3 is:

$$2x - 5 + 5 = 6 - 9x + 5$$

Step 5 The calculations in Step 4 are performed. ($5 - 5 = 0$ and $6 + 5 = 11$) to obtain the following equation.

$$2x = 11 - 9x$$

Step 6 $2x$ is isolated on the lhs of the equation and the like term, $9x$ must be moved to the lhs to combine with the $2x$. This move is accomplished by adding $9x$ to both sides as shown below.

$$2x + 9x = 11 - 9x + 9x$$

Step 7 $2x$ and $9x$ are like terms because they contain the same unknown. Therefore these two terms can be combined to obtain the following equation:

$$11x = 11$$

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Step 8 The solution is obtained as in previous lessons by dividing both sides by 11. The resulting equation is $^{11x}/_{11} = {}^{11}/_{11} \rightarrow x = 1$. Therefore, {1} is the solution set to $2x - 5 = 6 - 9x$.

Example Set 3

Find the solution to $3x = 18 - 6x$

Step 1 Look at the equation to see that "x" is the unknown quantity and there is an unknown (x) on both sides of the equal sign ($3x$ and $6x$).

Step 2 Looking at the Rule, we know that "x" must be isolated on one side of the equal sign so that a solution is obtained and therefore the like terms ($3x$ and $6x$) must be combined to accomplish this.

Step 3 3x is isolated on the lhs of the equation and the like term, $6x$ must be moved to the lhs to combine with the $3x$. This move is accomplished by adding 6x to both sides as shown below.

$$3x + 6x = 18 - 6x + 6x$$

Step 4 The calculations involving like terms in Step 3 are performed. ($3x + 6x = 9x$ and $6x - 6x = 0$) to obtain the following equation.

$$9x = 18$$

Step 5 The solution is obtained as in previous lessons by dividing both sides by 9. The resulting equation is $^{9x}/_{9} = {}^{18}/_{9} \rightarrow x = 2$. Therefore, {2} is the solution set to $3x = 18 - 6x$.

Apply what you have learned about solving equations containing more than one like term by working the activities for this lesson.

LIFE PRINCIPLE



"Adding positive factors to life cancels many negative features."

—Anonymous

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Basic Math Skills

Mathematics Charts

LENGTH - ENGLISH SYSTEM

12 inches	=	1 foot
3 feet	=	1 yard
5.5 yards	=	1 rod
320 rods	=	1 mile
1,760 yards	=	1 mile
5,280 feet	=	1 mile

VOLUME - ENGLISH SYSTEM

1 tablespoon, Tbsp, T	=	3 teaspoons, tsp
1 cup, c	=	16 tablespoons, Tbsp, T
1 pint, pt	=	2 cups, c
1 quart, qt	=	2 pints, pts
1 gallon, gal	=	4 quarts, qts

WEIGHT - ENGLISH SYSTEM

1 pound, lb	=	16 ounces, oz
1 hundredweight, cwt	=	100 pounds, lbs
1 ton	=	20 hundredweight, cwt
1 ton	=	2000 pounds, lbs

LENGTH - METRIC SYSTEM

1 meter	=	1,000 millimeters
1 meter	=	100 centimeters
1 kilometer	=	1,000 meters
1 centimeter	=	10 millimeters

VOLUME - METRIC SYSTEM

1 liter	=	1,000 milliliters
1 liter	=	100 centiliters
1 kiloliter	=	1,000 liters
1 centiliter	=	10 milliliters

WEIGHT - METRIC SYSTEM

1 gram	=	1,000 milligrams
1 gram	=	100 centigrams
1 kilogram	=	1,000 grams
1 centigram	=	10 milligrams

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Mathematics Charts

PERFECT SQUARES

$\sqrt{1}=1$	$\sqrt{25}=5$	$\sqrt{81}=9$	$\sqrt{169}=13$
$\sqrt{4}=2$	$\sqrt{36}=6$	$\sqrt{100}=10$	$\sqrt{196}=14$
$\sqrt{9}=3$	$\sqrt{49}=7$	$\sqrt{121}=11$	$\sqrt{225}=15$
$\sqrt{16}=4$	$\sqrt{64}=8$	$\sqrt{144}=12$	$\sqrt{256}=16$

PERFECT CUBES

$\sqrt[3]{1}=1$	$\sqrt[3]{-8}=-2$	$\sqrt[3]{64}=4$	$\sqrt[3]{-125}=-5$
$\sqrt[3]{-1}=-1$	$\sqrt[3]{27}=3$	$\sqrt[3]{-64}=-4$	$\sqrt[3]{216}=6$
$\sqrt[3]{8}=2$	$\sqrt[3]{-27}=-3$	$\sqrt[3]{125}=5$	$\sqrt[3]{-216}=-6$

$$P = \frac{\text{the number of elements in the set containing favorable outcomes}}{\text{the number of elements in the sample set}}$$

$$Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

FORMULAS

Surface Area	Area	Volume	Perimeter/ Circumference	Others
Sphere: $A_s = 4\pi r^2$	**Triangle:** $A = {}^1\!/_2 bh$	**Sphere:** $V = {}^4\!/_3 \pi r^3$	**Perimeter of a Rectangle:** $P = 2w + 2l$	**Pythagorean Theorem:** $c^2 = a^2 + b^2$
Cylinder: $A_s = 2\pi rh + 2\pi r^2$	**Rectangle:** $A = l \times w$	**Cone:** $V = {}^1\!/_3 \pi r^2 h$	**Circumference of a Circle:** $C = 2\pi r$ or $C = \pi d$	**Convert to Fahrenheit:** $F = {}^9\!/_5 C + 32$
Cube: $A_s = 6s^2$	**Circle:** $A = \pi r^2$	**Rectangular Prism:** $V = l \times w \times h$		**Convert to Celsius:** $C = {}^5\!/_9 (F - 32)$
Lateral Surface Area of a Cone: $A_{ls} = \pi rl$	**Trapezoid:** $A = {}^1\!/_2 (b_1 + b_2)h$	**Square Pyramid:** $V = ({}^1\!/_3) lwh$		**Distance:** $d = r \bullet t$
		Triangular Prism: $V = {}^1\!/_2 bhl$		**Slope:** $y = mx = b$
		Cylinder: $V = \pi r^2 h$		**Simple Interest:** $I = prt$

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Mathematics Charts

TIME CONVERSIONS

1 week = 7 days
1 year = 12 months

CONVERSIONS

English to Metric	
1 ounce (oz)	28.35 grams (g)
1 pound (lb)	454 grams (g)
1 ton (T)	907.2 kilograms (kg)

Metric to English	
1 gram (g)	.035 ounce (oz)
1 kilogram (kg)	2.2 pounds (lb)
1 metric ton (mt)	2200 pounds (lb)

English to Metric	
1 cubic inch	16.39 cubic cm
1 gallon	3.785 liters
1 cubic foot	28.316 liters
1 quart	.946 liters

Metric to English	
1 cubic cm	.061 cubic inch
1 cubic meter	35.315 cubic feet
1 liter	1.05 quarts
1 liter	.264 gallon

English to Metric	
1 inch	2.54 cm
1 foot	0.3048 meter
1 yard	0.9144 meter
1 mile	1.6 km

Metric to English	
1 centimeter	0.3937 in.
1 meter	39.37 in.
1 meter	1.09 yds
1 kilometer	0.6213 mile

AREA CONVERSIONS

1 square centimeter = 0.155000 sq in.
1 square meter = 10.763910 sq ft
1 square inch = 645.16 sq mm
1 square foot = 0.09290304 sq meter
1 square yard = 0.836127 sq meter
1 square foot = 144 sq in.

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